第 8 章 習題簡答

習題 8-1

1.(1) $D_f = \mathbb{R}^2 \setminus \{(0,0)\}$ (2) $D_f = (\mathbb{R} \setminus \{0\}) \times (\mathbb{R} \setminus \{0\})$

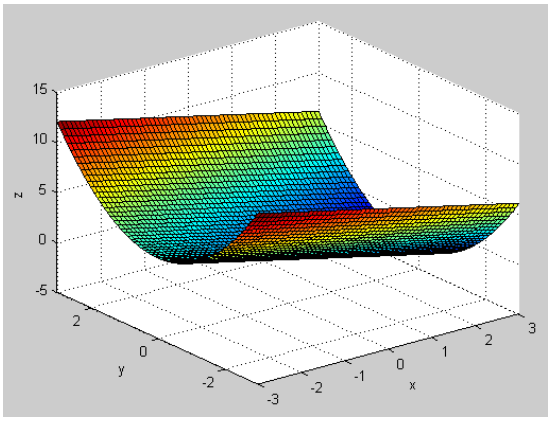
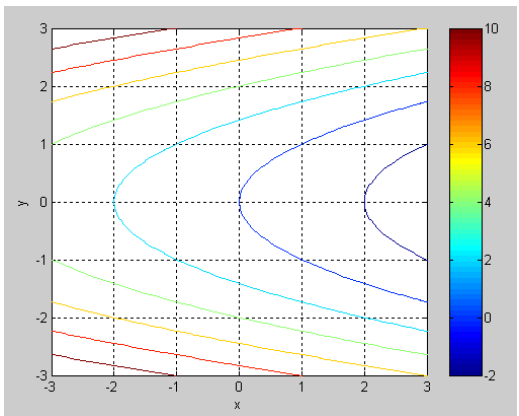
(3) $D_f = \{(x,y) \big| \, |x| \geq |y|, x, y \in \mathbb{R}\}$ (4) $D_f = \{(x,y) \big| \, x^2 + y^2 < 9, x, y \in \mathbb{R}\}$

(5) $D_f = \{(x,y) \big| \, x^2 + y^2 \neq 1\}$ (6) $D_f = \{(x,y,z) \big| \, x^2 + y^2 + z^2 \geq 5^2\}$

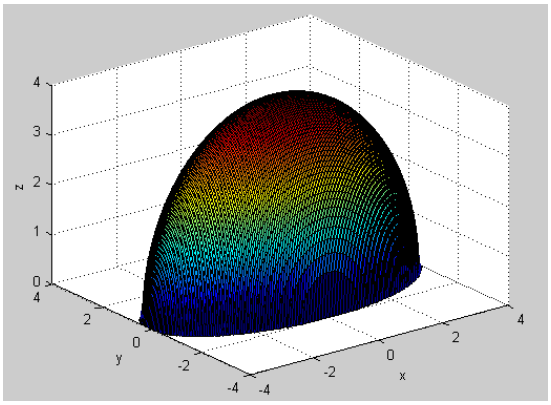
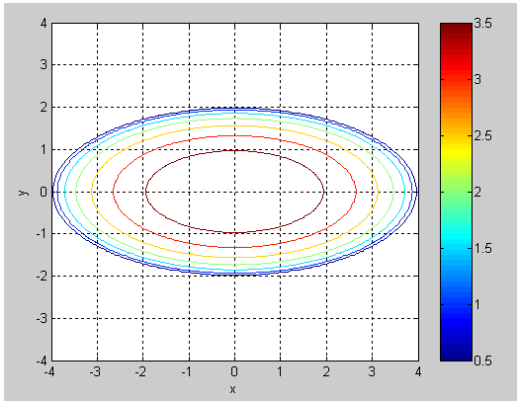
(7) $D_f = \{(x,y,z) \big| \, |x| \neq |z|\}$ (8) $D_f = \{(x,y,z) \big| \, x^2 + y^2 \leq 16, z \neq 4\}$

2.(1) $R_f = [10, \infty)$ (2) $R_f = [0,6]$ (3) $R_f = [-1,1]$ (4) $R_f = (-\infty, 6]$

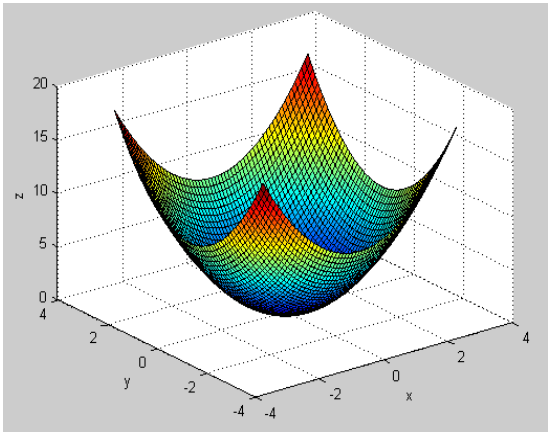
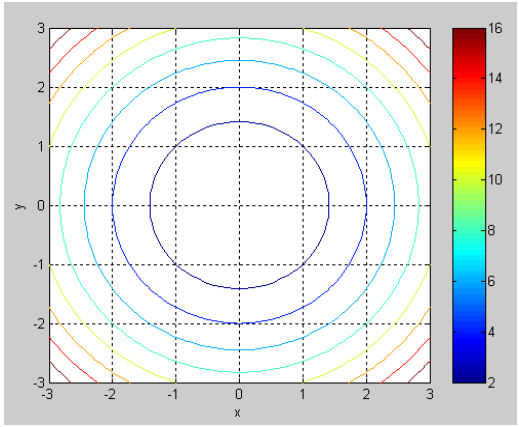
3.(1) $f(x,y) = y^2 - x$

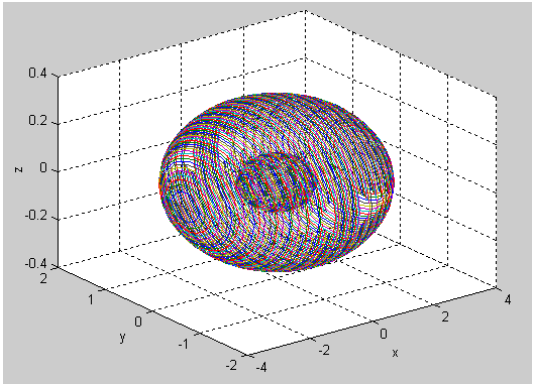
(2) $f(x,y) = \sqrt{16 - x^2 - 4y^2}$

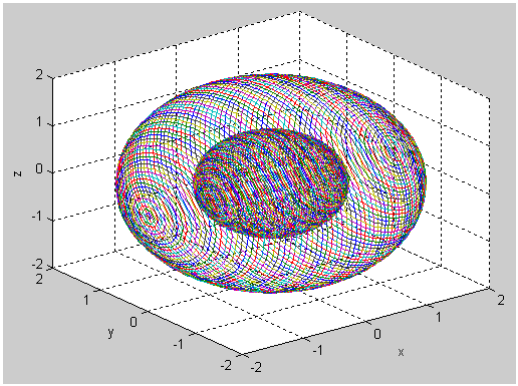
(3) $f(x,y) = x^2 + y^2$

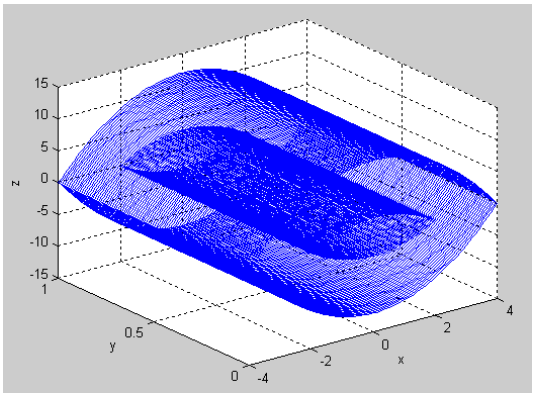
4.(1) $f(x,y,z)=x^2+4y^2+9z^2$



(2) $f(x,y,z)=x^2+y^2+z^2$



(3) $f(x,y,z)=\frac{x^2}{4}+\frac{z^2}{9}$



習題 8-2

1.(1)$\frac{6}{5}$ (2) 5 (3)不存在 (4)不存在 (5) 0 (6) 0 2.(1)是 (2)是

(3)否，∵ $f(0,0)$ 無定義 (4)否，∵ $\{(x,y)\,|\,2x+y=0\}$ 不連續

(5)否，因只在 $\{(x,y)\,|\,x^2+y^2<4\}$ 連續。(6)否，因只在 $\{(x,y)\,|\,y\ge 0, x+y\ge 0\}$ 連續

3.(1)是 (2)否 (3)是 4. $f(0,0)=0$

習題 8-3

1. $f_x=3x^2+14xy+3,\ f_y=7x^2+24y^2-2$ 2. $\frac{\partial z}{\partial x}=\tan^{-1}\frac{y}{x}-\frac{xy}{x^2+y^2}, \frac{\partial z}{\partial y}=\frac{x^2}{x^2+y^2}$

3. $f_x(0,\frac{\pi}{4})=-1,\ f_y(0,\frac{\pi}{4})=0$ 4.略 5. $\frac{\partial f}{\partial z}=(xy)^{\sin z}\cdot\cos z\cdot\ln(xy)$

6. $\frac{\partial z}{\partial x}=\frac{yz^2-2x}{y^2-2xyz}, \frac{\partial z}{\partial y}=\frac{xz^2-2yz}{y^2-2xyz}$ 7. 2 8. 0 9. $\frac{\partial R}{\partial R_3}=\left(\frac{R_1R_2}{R_2R_3+R_1R_3+R_1R_2}\right)^2$

10.(1) $f_x(0,0)=0,\ f_y(0,0)=0$ (2) $f_x(0,0)=1,\ f_y(0,0)=1$

11. $f_{xxy}=\frac{6x^2y-2y^3-2yz^2}{(x^2+y^2+z^2)^3},\ f_{yxx}=f_{xxy}$ 12. $f_{xx}=6x+4y$

13. $f_{xy}=e^{-xy}(y\sin x-\cos x),\ f_{yx}=f_{xy}$ 14. $f_{xy}=ze^{yz}+ze^{xz}+(1+xy)ze^{xy}$ ，

$f_{yxz}=(1+yz)e^{yz}+(1+xz)e^{xz}+(1+xy)e^{xy}$ ， $f_{zxy}=f_{yxz}$

15. 切線斜率：–2，切線方程式：$\begin{cases} z-1=-2(y-2) \\ x=2 \end{cases}$

16. 切線方程式：$\begin{cases} z+3=2(x+1) \\ y=2 \end{cases}$ 17.略 18.略

習題 8-4

1. $\frac{\partial u}{\partial s}=2xe^t+2yte^s, \frac{\partial u}{\partial t}=2xse^t+2ye^s$ 2. $\frac{du}{dt}=-\frac{1}{t^2}(y+z)+(x+z)e^t-(y+x)e^{-t}$

3.略 4. $\frac{\partial u}{\partial s}=(1+y^2z)2s+(z^2+2xyz)t+(2yz+xy^2)2s$ ，

$\frac{\partial u}{\partial t}=(1+y^2z)2t+(z^2+2xyz)s-(2yz+xy^2)2t$ 5. $\frac{du}{dx}=\frac{y-xe^x}{y^2+x^2}$

6. $\frac{du}{dx}=\frac{x+y\sin x+xy\cos x+z\sec^2 x}{\sqrt{x^2+y^2+z^2}}$ 7. $\frac{\partial z}{\partial \theta}\Big|_{r=3,\theta=\pi/6}=\frac{1}{2}, \frac{\partial z}{\partial r}\Big|_{r=3,\theta=\pi/6}=0$ 8.略 9.略

10. 14 吋/秒 11.體積以 306 呎^3/秒 的速度增加。 12. 44π 吋^2/分

13. $\frac{dy}{dx}=-\frac{2x+3y}{3x-4y}$ 14. $\frac{\partial z}{\partial x}\Big|_{(0,0,2)}=0, \frac{\partial z}{\partial y}\Big|_{(0,0,2)}=\frac{1}{3}$ 15. $\frac{dy}{dx}=\frac{y}{x-3y^4-3y^2x^2}$

16.(題目更改)設 $xe^{yz}+ye^{xz}-y^2+3=0$，求 $\frac{dz}{dx}$ 及 $\frac{dz}{dy}$。

$\frac{\partial z}{\partial x}=-\frac{e^{yz}+yze^{xy}}{xye^{yz}+xye^{xz}}, \frac{\partial z}{\partial y}=-\frac{xze^{yz}+e^{xz}-2y}{xye^{yz}+xye^{xz}}$

17. $\frac{dy}{dx}=\frac{-2y-z-x}{z-y}, \frac{dz}{dx}=\frac{x+2z+y}{z-y}$ 18. $\frac{dy}{dx}=\frac{-xy^2-2x+z}{2x^2y+2y}, \frac{dz}{dx}=\frac{-2xyz-2y^3-4y}{2x^2y+2y}$

習題 8-5

1. $\Delta f=0.508, df=0.5$ 2. $\Delta z=-0.87, dz=-0.9$ 3.(1) $dz=\frac{3x^2dx-4ydy}{\sqrt{2x^3-4y^2}}$

(2) $dz=\frac{ydx-xdy}{y\sqrt{y^2-x^2}}$ (3) $dz=(3x^2+2x^4)e^{x^2-y^2}dx-2x^3ye^{x^2-y^2}dy$

(4) $du=z\cos(y+z)dy+[\sin(y+z)+z\cos(y+z)]dz$

(5) $du=\frac{-xzdx-yzdy+(x^2+y^2)dz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$

(6) $du=[\frac{1}{x}-z\sin(xz)]dx+[\frac{1}{y}+ze^{yz}]dy+[ye^{yz}-x\sin(xz)]dz$

4. $df=(2x+y)^{x-y}\left\{[\ln(2x+y)+\frac{2(x-y)}{2x+y}]dx+[-\ln(2x+y)+\frac{x-y}{2x+y}]dy\right\}$

5. 10.004 6. 12.985 7. 0.39608 8. 0.11 呎^2

9.近似值：5.2865，誤差：0.0903，百分誤差：1.738% 10. 0.07850 立方公尺

11.(a) 37.398 cm^3 (b) 3.770 cm^2 12. –0.5% 13.近似誤差：0.01482，百分誤差：0.42% 14.最大誤差：–0.0002，百分誤差：–0.00166%

15. $96000\ (\text{cm}\cdot\text{rad}^2/\text{min}^2)$

習題 8-6

1.(1) $\nabla f=\langle 3x^2-2xy^2,-2x^2y+3y^2\rangle$ (2) $\nabla f=\langle -4xy^3e^{-2x^2y^3},1-6x^2y^2e^{-2x^2y^3}\rangle$

(3) $\nabla f=\langle \frac{2xy^3}{z^4},\frac{3x^2y^2}{z^4},\frac{-4x^2y^3}{z^5}\rangle$ (4) $\nabla f=\langle \frac{x}{\sqrt{x^2+y^2z^2}},\frac{yz^2}{\sqrt{x^2+y^2z^2}},\frac{y^2z}{\sqrt{x^2+y^2z^2}}\rangle$

2. $\langle -12,-9\rangle$ 3. $\langle \frac{4}{25},\frac{-3}{25},\frac{1}{5}\rangle$ 4. $\frac{136}{13}$ 5. -1 6. $\frac{-7}{\sqrt{17}}$ 7. $\frac{1}{3}$ 8. $\sqrt{409}$ 9. $\sqrt{41}$

10.方向應指向 $\langle 5,5\rangle$，此時方向導數的最大值為 $5\sqrt{2}$。

習題 8-7

1.切平面方程式：$2x-y+z-9=0$，法線：$\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z-4}{1}$

2.切平面方程式：$4x+5y-2z-16=0$，法線：$\frac{x-1}{4}=\frac{y-2}{5}=\frac{z+1}{-2}$

3.切平面方程式：$6x-2y+15z-22=0$，法線：$\frac{x-1}{6}=\frac{y-7}{-2}=\frac{z-2}{15}$

4.切平面方程式：$3x+3y+2z-10=0$，法線：$\frac{x-1}{3}=\frac{y-1}{3}=\frac{z-2}{2}$

5.切平面方程式：$8x-5y-6z=0$，法線：$\frac{x-1}{8}=\frac{y+2}{-5}=\frac{z-3}{-6}$

6.切平面方程式：$4x-8y-z-8=0$，法線：$\frac{x-2}{4}=\frac{y+1}{-8}=\frac{z-8}{-1}$

7.切平面方程式：$2x+2y-4z-\pi=0$，法線：$\frac{x-1}{1}=\frac{y+1}{1}=\frac{z+\frac{\pi}{4}}{-2}$

8.切線：$\begin{cases}x=3\\ y=4t\\ z=t\end{cases},t\in\mathbb{R}$，法平面方程式：$4y+z=0$ 9.略 10.略 11.略 12.略

13.略 14.切線：$\begin{cases}x=4-t\\ y=-1\\ z=t\end{cases},t\in\mathbb{R}$ 15.略 16.略

習題 8-8

1.(1)極小值：$f(1,2)=-2$ (2)鞍點：$(1,1),(-1,-1)$ (3)極大值：$f(\frac{2}{3},\frac{4}{3})=\frac{59}{27}$，鞍點：$(0,0),(0,4),(2,0)$ (4)鞍點：$(2,-3)$ (5)無極值及鞍點 (6)極大值：$f(0,0)=1$，極小值：$f(\pm\frac{1}{\sqrt{2}},\pm\frac{1}{\sqrt{2}})=\frac{1}{2}$，鞍點：$(0,\pm\frac{1}{\sqrt{2}}),(\pm\frac{1}{\sqrt{2}},0)$

2. $(\pm\sqrt{2},-1,\sqrt{2}),(\pm\sqrt{2},-1,-\sqrt{2})$ 3.當長、寬、高均為 $\sqrt{\frac{A}{6}}$ 時有最大體積 $\frac{A\sqrt{6A}}{36}$

4.長：寬：高 $=2:2:1$ 5. $x=\frac{a}{\sqrt{3}}, y=\frac{b}{\sqrt{3}}, z=\frac{c}{\sqrt{3}}$ 時有最大體積 $\frac{8\sqrt{3}abc}{9}$

6.當 $x=y=\frac{12}{5}, z=\frac{6}{5}$ 時有最大值 $\frac{124416}{3125}$

7.(題目更正)設 x, y, z 為三正數，且滿足 $2xy+3yz+xz=72$，試求 xyz 之極大值。

解：當 $x=6, y=2, z=4$ 時有極大值 48 8. $P(\frac{1}{2}, \frac{1}{2})$ 9.略 10. $(8,0,1)$ 11. $\frac{1}{2}$

12.當長、寬、高均為 $\frac{a}{3}$ 時有最大體積 $\frac{a^3}{27}$ 13.(1) $y=0.76x+0.43$

(2) $y=1.13x-0.71$ (3) $y=0.39x+3.21$ 14. $y=-1.54x+97.10$

15. $y=-12.75x+301.5$