第 7 章 習題簡答

習題 7-1

1. （1）$\overline{PQ} = 2\sqrt{6}$，中點：$(0,0,4)$　（2）$\overline{PQ} = \sqrt{26}$，中點：$(-\frac{3}{2}, \frac{3}{2}, 2)$

2. $P(2\sqrt{3}, 2\sqrt{3}, 2\sqrt{3})$ 或 $P(-2\sqrt{3}, -2\sqrt{3}, -2\sqrt{3})$　3. 略　4. $P(-\frac{7}{3}, -\frac{1}{3}, 3)$　5. $C(2,0,9)$

習題 7-2

1.(1)$\langle -11, 2, 24 \rangle$　(2)-20　(3)$\langle 54, 39, -6 \rangle$　(4)$\langle -23, -38, -28 \rangle$　(5)$\langle -35, -80, -10 \rangle$
(6)15　2.略　3.略　4.略

習題 7-3

1. $\begin{cases} x = & 2+2t \\ y = & 3+4t, t \in \mathbb{R} \\ z = & -1+3t \end{cases}$　2. $\dfrac{x-1}{3} = \dfrac{y-2}{-5} = \dfrac{z-3}{-2}$　3.參數式：$\begin{cases} x = & 3-t \\ y = & -1+4t, t \in \mathbb{R}, \\ z = & 2-7t \end{cases}$　對稱

式：$\dfrac{x-3}{-1} = \dfrac{y+1}{4} = \dfrac{z-2}{-7}$　4.$(2,-2,-1)$　5.$\cos^{-1}\dfrac{4}{21}$ 或 $\pi - \cos^{-1}\dfrac{4}{21}$

6. $\begin{cases} x = & 1-12t \\ y = & -2+4t, t \in \mathbb{R} \\ z = & 5+3t \end{cases}$　7. $5x - y + z + 5 = 0$　8. $x - y - 2z = 8$　9. $3x - 2y - 7z = 0$

10. $51x - 58y + 47z = 118$　11. $12x + 4y - 3z + 26 = 0$ 或 $12x + 4y - 3z = 0$

習題 7-4

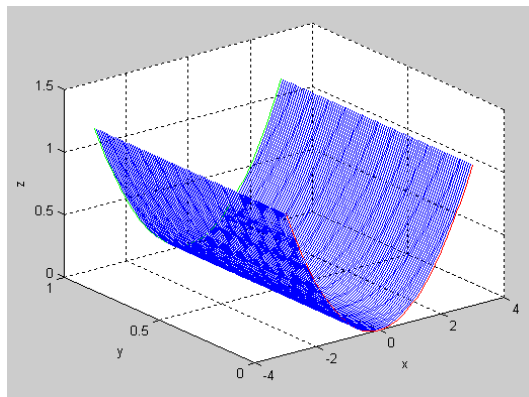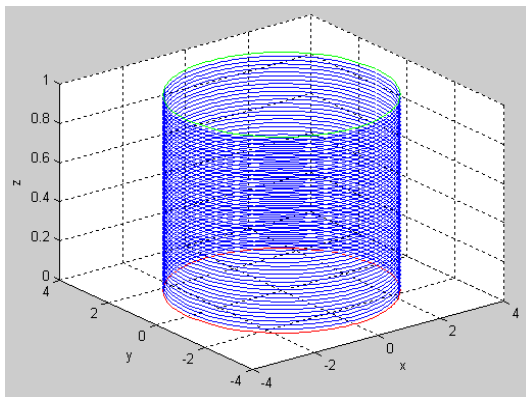1.$(x-2)^2 + (y-1)^2 + z^2 = 19$　2.$x^2 + y^2 + z^2 - x - y - z = 0$　3.$x^2 + (y-3)^2 + z^2 = 25$

或 $x^2 + (y+7)^2 + z^2 = 25$　4.$x^2 + y^2 + (z-\dfrac{13}{10})^2 = \dfrac{2669}{100}$

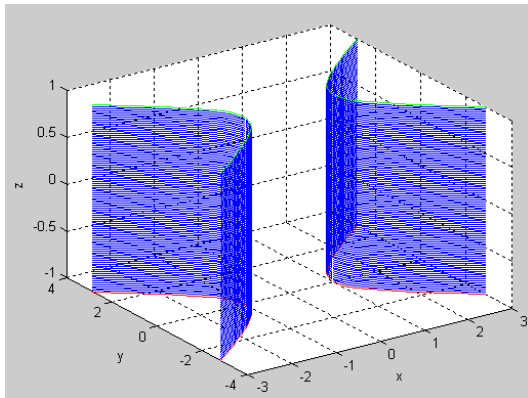5.$(x-1)^2 + (y+1)^2 + (z-2)^2 = \dfrac{32}{3}$　6.$x - y - \sqrt{2}z - 2 = 0$ 或 $x - y + \sqrt{2}z - 2 = 0$

7.$r = \dfrac{\sqrt{210}}{6}$，切點：$(\dfrac{5}{6}, -\dfrac{4}{3}, \dfrac{11}{6})$　8.$\dfrac{275}{9}\pi$
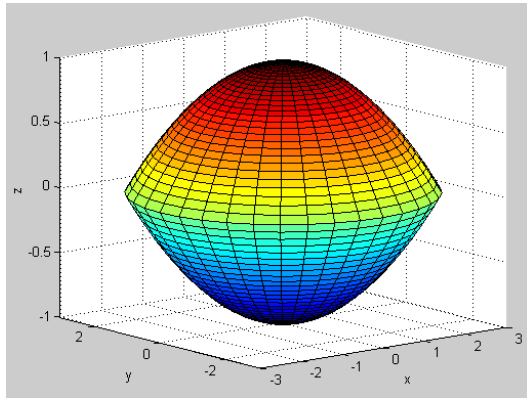
9. (1) $x^2 + y^2 = 9$　　　　　　　　　　　(2) $x^2 = 8z$
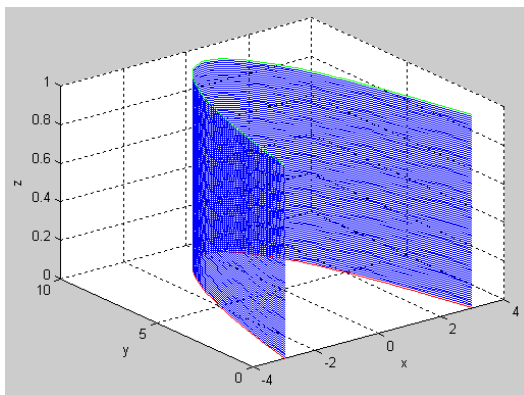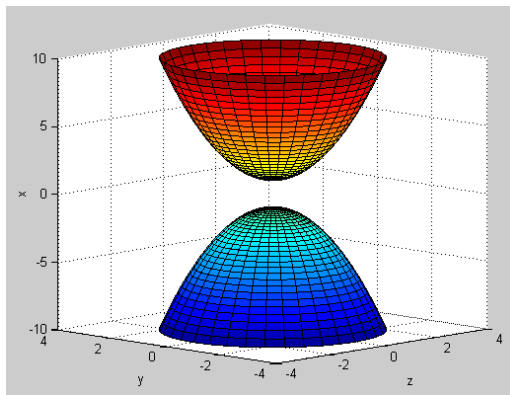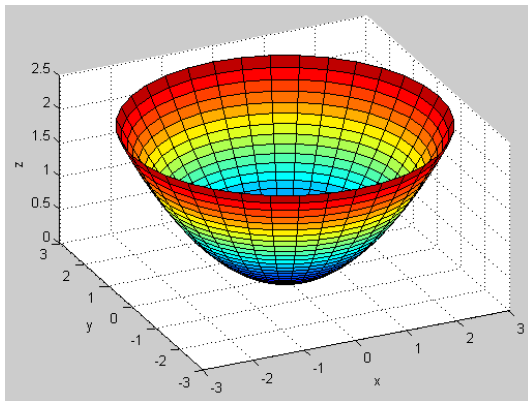
(3) $x^2 - y^2 = 1$

(4) $x^2 + y^2 + 9z^2 = 9$

(5) $y = 9 - x^2$
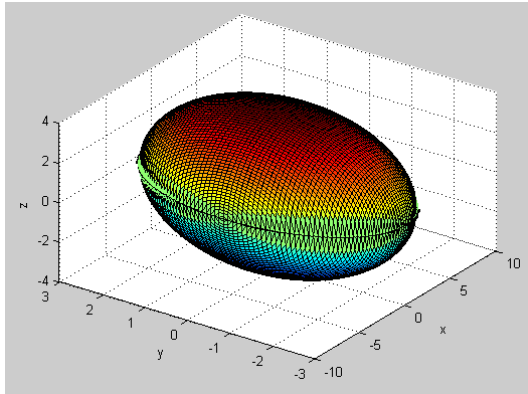

(6) $x^2 - y^2 - z^2 = 1$

(7) $x^2 + y^2 = 4z$

10.(a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{b^2} = 1$ (b) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{a^2} = 1$ 11.(a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} - \dfrac{z^2}{b^2} = 1$

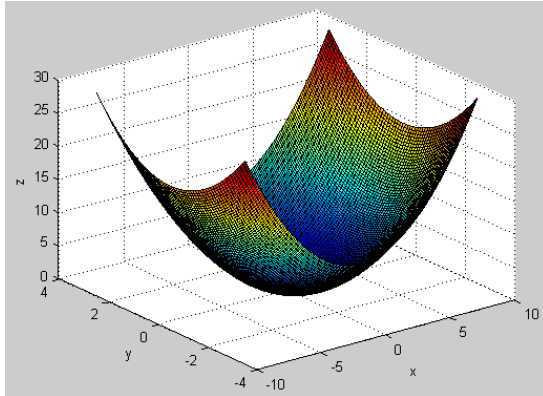(b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} - \dfrac{z^2}{a^2} = 1$
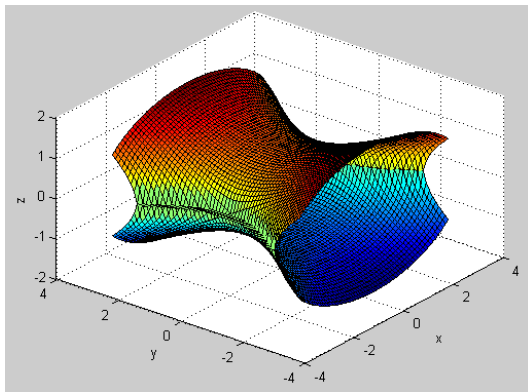
12. $y^2 + z^2 = 4cx$

13. $|y| = m\sqrt{x^2 + z^2}$

14. (1)橢圓面：$\dfrac{x^2}{6^2}+\dfrac{y^2}{3^2}+\dfrac{z^2}{2^2}=1$　　　(2)橢圓拋物面：$\dfrac{x^2}{2^2}+\dfrac{y^2}{1^2}=z$ (題目更改)
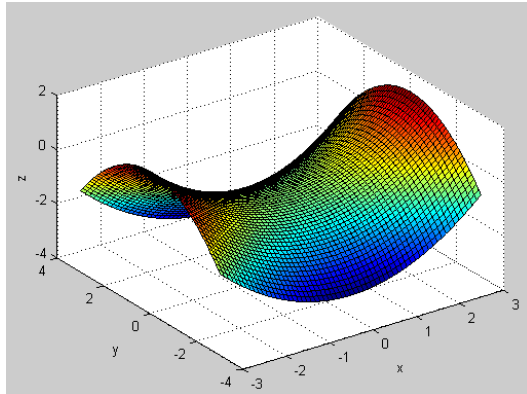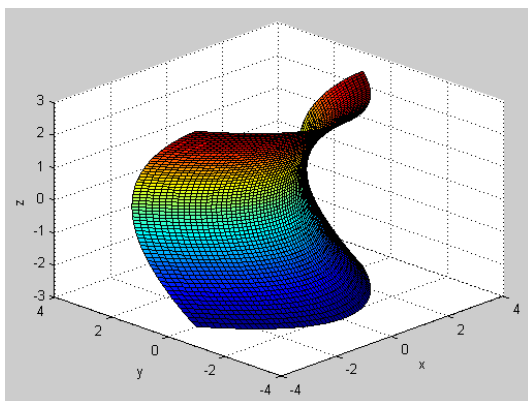

(3)單葉雙曲面：$\dfrac{x^2}{2^2}-\dfrac{y^2}{2^2}+\dfrac{z^2}{1^2}=1$　　　(4)雙曲拋物面：$\dfrac{x^2}{2^2}-\dfrac{y^2}{2^2}=z+1$
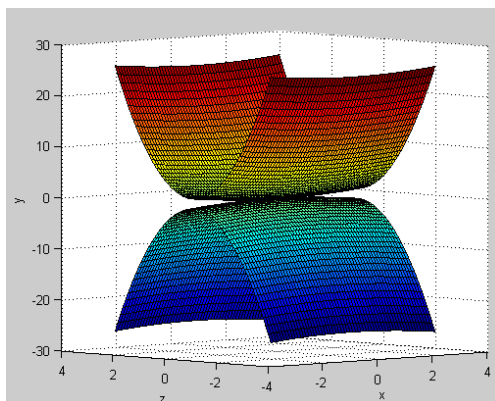

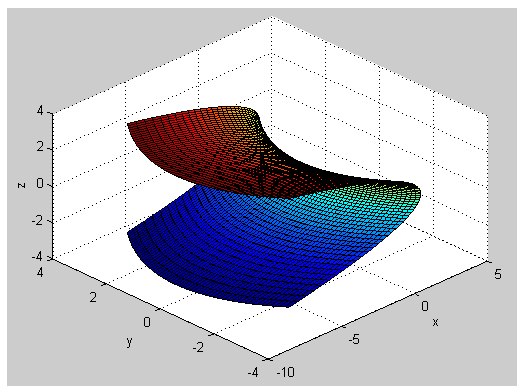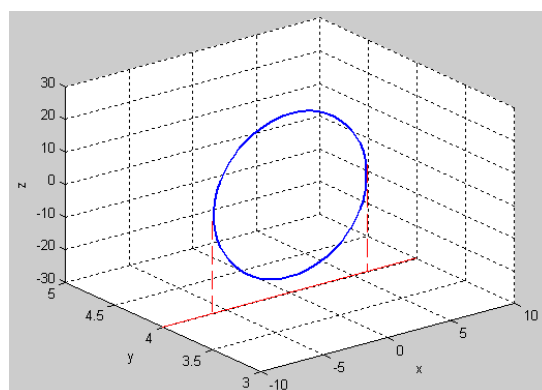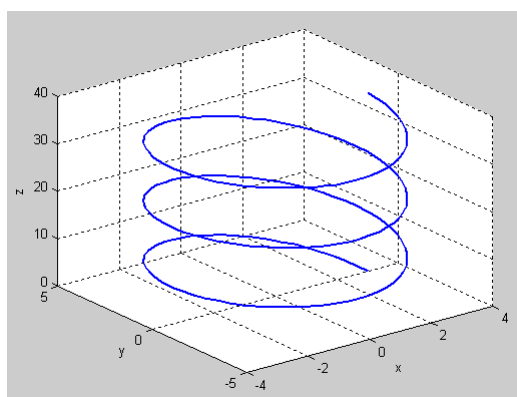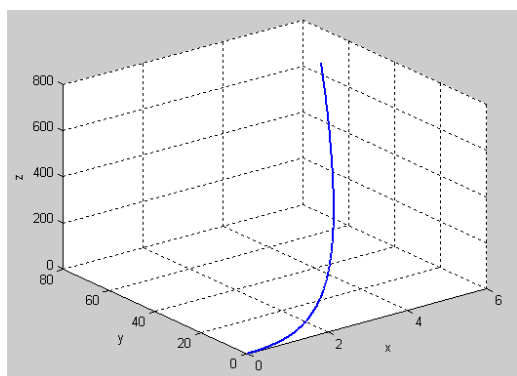(5)雙曲拋物面：$\dfrac{x^2}{2^2}-\dfrac{z^2}{2^2}=y$　　　(6)橢圓錐面：$x^2-9y^2+25z^2=0$

(7)雙曲拋物面：

15.(1) $x = 3\cos t, y = 5\sin t, z = 2t, t \geq 0$　　(2) $x = 6\sin t, y = 4, z = 25\cos t, t \geq 0$

(3) $x = t, y = 2t^2, z = 3t^3, t \geq 0$

習題 7-5

1.(1)柱面座標：$(2\sqrt{5}, \tan^{-1}\frac{1}{2}, -4)$，球面座標：$(6, \pi - \cos^{-1}\frac{2}{3}, \tan^{-1}\frac{1}{2})$

(2)柱面座標：$(2, \frac{5\pi}{3}, 4)$，球面座標：$(2\sqrt{5}, \cos^{-1}\frac{2}{\sqrt{5}}, \frac{5\pi}{3})$

(3)柱面座標：$(\sqrt{2}, \frac{\pi}{4}, 1)$，球面座標：$(\sqrt{3}, \cos^{-1}\frac{\sqrt{3}}{3}, \frac{\pi}{4})$

2. $(0, 10, 4)$　3. $(1, 1, \sqrt{2})$　4. $(2\sqrt{10}, \cos^{-1}\frac{3}{\sqrt{10}}, \frac{2\pi}{3})$　5.(1) $r^2(1 + \sin 2\theta) = z - 5$

$(2) \dfrac{r^2 \cos^2 \theta}{a^2} + \dfrac{r^2 \sin^2 \theta}{b^2} = 1$ $(3) ar \cos \theta + br \sin \theta + cz = r^2 + z^2$ $6.(1) z = x + y$

$(2) z^2 - x^2 - y^2 = a^2$ $(3) x^2 + y^2 + z^2 = ay$ $(4) 4x^2 y^2 (x^2 + y^2 + z^2) = a^4 (x^2 + y^2)$

习题 7-6

$1.(1) \langle -\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}, -1 \rangle$ $(2) \langle 2e, 6, \dfrac{1}{3} \rangle$ $2.(1) \langle \dfrac{1}{4}, \dfrac{5}{8}, e - 1 \rangle$ $(2) \langle \dfrac{1}{2}(e-1), \dfrac{2}{3}, -\dfrac{2}{3} \rangle$ 3.略

4.略