

第 5 章 習題簡答

習題 5-1

1. (1) $\frac{1}{20}(2x+1)^5 - \frac{1}{16}(2x+1)^4 + c$ (2) $\frac{1}{36}(x^4+1)^9 + c$ (3) $-\frac{1}{15(x^3+2)^5} + c$ (4) $\frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$
 (5) $\frac{1}{8}\sqrt{1+8x^2} + c$ (6) $\frac{1}{5}(x+2)^5 - \frac{1}{2}(x+2)^4 + c$ (7) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$
 (8) $\frac{1}{11}(x^3+1)^{\frac{11}{3}} - \frac{1}{8}(x^3+1)^{\frac{8}{3}} + c$ (9) $\frac{2}{3}(1+x)^{\frac{3}{2}} - 2(1+x)^{\frac{1}{2}} + c$ (10) $\frac{1}{3}(1+x^2)^{\frac{3}{2}} - (1+x^2)^{\frac{1}{2}} + c$
2. (1) $-\frac{2^{11}}{11}$ (2) 513 (3) $\sqrt{18} - \sqrt{11}$ (4) $\frac{2}{3}(\ln 22 - \ln 3)$ (5) $\frac{1}{3\sqrt{2}}$ (6) $\frac{3392}{2835}$ (7) $\frac{506}{375}$

3. 略 4. 略

習題 5-2

1. $2(e^2 - e)$ 2. $\frac{1}{2}(e^5 - e^2)$ 3. $\frac{1}{4}(\ln(\ln x^2))^2 + c$ 4. $-\frac{1}{e^x+1} + c$ 5. $x - \ln(1+e^x) + c$
 6. $-\frac{1}{6}\ln|1-2x^3| + c$ 7. $-\frac{1}{6}e^{-3x^2} + c$ 8. $-e^{\frac{1}{x}} + c$ 9. $\ln(1+e^x) + c$ 10. $2e^{\sqrt{x}} + c$
 11. $\frac{1}{2}\ln|2e^x - 1| + c$ 12. $e^{e^x} + c$ 13. $\ln(e^x + e^{-x}) + c$
 14. 題目更改： $\int \frac{dx}{e^x - 2 + e^{-x}} = -\frac{1}{e^x - 1} + c$ 15. $2\sqrt{e^x - 1} + c$ 16. $e^x - 3\ln(e^x + 1) + c$
 17. $2\ln|1+e^{\sqrt{x}}| + c$ 18. $\frac{1}{3}[\ln(1+e^x)]^3 + c$ 19. $\frac{3^{\ln 2x}}{\ln 3} + c$

習題 5-3

1. $\ln|\sin x| + c$ 2. $\ln|\csc x - \cot x| + c$ 3. $\frac{1}{5}\ln|\sec 5x| + c$ 4. $\frac{1}{5}\ln|3 + \sin 5x| + c$ 5. $x + \cos x + c$
 6. $-\frac{2}{3}\cot 3x - x - \frac{2}{3}\csc 3x + c$ 7. $\frac{2}{3}(5 + \tan x)^{\frac{3}{2}} + c$ 8. $-\frac{1}{6}(\sin x + \cos x)^6 + c$ 9. $\frac{1}{2}[\ln(\sin x)]^2 + c$
 10. $\frac{2}{9}(4 + 3\sec x)^{\frac{3}{2}} + c$ 11. $\frac{\pi^2}{32}$ 12. $(\tan^{-1}\sqrt{x})^2 + c$ 13. $e^{4+\sin x} + c$ 14. $\frac{1}{3}\cos(\cos 3x) + c$
 15. $\sin e^x + c$ 16. $\tan^{-1} e^x + c$ 17. $\ln|e^x - e^{-x}| + c$ 18. $-2\cos\sqrt{x} + c$ 19. $\sin(\ln x) + c$
 20. $-e^{\cos x} + c$ 21. $\frac{1}{2}(\tan^{-1} x)^2 + c$ 22. $\frac{1}{2}(\sin^{-1} x)^2 + c$ 23. $\sin^{-1}\frac{\ln x}{3} + c$ 24. $\frac{1}{2}\sec^{-1} x^2 + c$
 25. $\frac{1}{3}\sin^{-1} x^3 + c$ 26. $\frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} + c$ 27. $\frac{1}{3}\sec^{-1}\frac{2x}{3} + c$ 28. $\frac{1}{2}\tan^{-1}\frac{\ln x}{2} + c$

習題 5-4

1. $-x \cos x + \sin x + c$ 2. $x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$ 3. $\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + c$
4. $-\frac{\ln x}{4x^4} - \frac{1}{16x^4} + c$ 5. $x \sin^{-1} 3x + \frac{1}{3} \sqrt{1-9x^2} + c$ 6. $x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + c$
7. $\frac{x^2}{3} e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c$ 8. $\frac{x^4}{4} \ln 5x - \frac{x^4}{16} + c$ 9. $x(\ln x)^2 - 2x \ln x + 2x + c$
10. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + c$ 11. $-\frac{x}{4} \cos 2x + \frac{1}{8} \sin 2x + c$ 12. $\frac{1}{3} x^3 e^{x^3} - \frac{1}{3} e^{x^3} + c$
13. $\frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + c$ 14. $\frac{1}{2} - \frac{1}{2} x^2 e^{-x^2} - \frac{1}{2} e^{-x^2} + c$ 15. $\frac{1}{2} x^2 \sin x^2 + \frac{1}{2} \cos x^2 + c$
16. $-\frac{1}{3} e^{6x} \cos e^{3x} + \frac{2}{3} e^{3x} \sin e^{3x} + \frac{2}{3} \cos e^{3x} + c$ 17. $\frac{e^{x^2}}{2(x^2+1)} + c$ 18. $\frac{1}{2} e^x (\sin x + \cos x) + c$
19. $\frac{1}{2} e^{\sin^{-1} x} (x + \sqrt{1-x^2}) + c$ 20. $x^2 e^x - 2x e^x + 2e^x + c$ 21. $\frac{x^2}{2} \sin^{-1} x - \frac{1}{4} \sin^{-1} x + \frac{x\sqrt{1-x^2}}{4} + c$
22. $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c$, 23. $\frac{x^4}{4} \ln x - \frac{x^4}{16} + c$ 24. $x \tan x - \ln |\sec x| + c$ 25. $\frac{x^2}{4} - \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + c$
26. $\frac{x^3}{3} \sin^{-1} x - \frac{1}{9} (1-x^2)^{\frac{3}{2}} + \frac{1}{3} (1-x^2)^{\frac{1}{2}} + c$ 27. $x^2 \sqrt{1+x^2} - \frac{2}{3} (1+x^2)^{\frac{3}{2}} + c$
28. $x^2 \sin x + 2x \cos x - 2 \sin x + c$ 29. $x \sin^{-1} x + \sqrt{1-x^2} + c$ 30. $x \ln x - x + c$
31. $\frac{2}{13} e^{2x} \cos 3x + \frac{3}{13} e^{2x} \sin 3x + c$ 32. $\frac{2}{13} e^{2x} \sin 3x - \frac{3}{13} e^{2x} \cos 3x + c$
33. $-\frac{1}{5} e^{-x} \cos 2x + \frac{2}{5} e^{-x} \sin 2x + c$ 34. $-\frac{1}{5} e^{-x} \sin 2x - \frac{2}{5} e^{-x} \cos 2x + c$
35. $\frac{2}{5} e^x \sin 2x + \frac{1}{5} e^x \cos 2x + c$ 36. $e^{\sin x} \sin x - e^{\sin x} + c$ 37. $\frac{x3^x}{\ln 3} - \frac{3^x}{(\ln 3)^2} + c$
38. $x \ln(1+x^3) - 3x + \ln|x+1| - \frac{1}{2} \ln(x^2-x+1) + \sqrt{3} \tan^{-1} \frac{2x-1}{\sqrt{3}} + c$ 39. $-\frac{1}{2} x^2 \cos x^2 + \frac{1}{2} \sin x^2 + c$
40. 略, 41 略, 42. $-\frac{12}{e^2} + 4$ 43. $\frac{\pi}{4} - \frac{1}{2}$ 44. $bf'(b) - af'(a) - f(b) + f(a)$

習題 5-5

1. $-\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x + c$ 2. $-\frac{1}{16} \cos 8x + \frac{1}{4} \cos 2x + c$ 3. $-\frac{1}{10} \sin 5x + \frac{1}{2} \sin x + c$
4. $\frac{1}{12} \sin 6x + \frac{1}{4} \sin 2x + c$ 5. $\frac{1}{18} \sin 9x + \frac{1}{6} \sin 3x + c$ 6. $-\frac{1}{18} \sin(9x+6) + \frac{1}{10} \sin 5x + c$
7. $\frac{1}{3} \cos^3 x - \cos x + c$ 8. $\frac{1}{2} (x - \sin x \cos x) + c$ 9. $\sin x - \frac{1}{3} \sin^3 x + c$

10. $-\left(\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x\right) + c$ 11. $\frac{5}{16}x - \frac{1}{4}\sin 2x + \frac{3}{64}\sin 4x + \frac{1}{48}\sin^3 2x + c$
 12. $x - \frac{1}{2}\cos 2x + c$, 13. $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + c$ 14. $\frac{x}{8} - \frac{1}{32}\sin 4x + c$
 15. $\frac{1}{9}\sin^9 x - \frac{2}{7}\sin^7 x + \frac{1}{5}\sin^5 x + c$ 16. $\frac{3}{2}x + \frac{3}{20}\sin 10x + c$ 17. $\frac{2}{9}\cos^{\frac{9}{2}} x - \frac{2}{5}\cos^{\frac{5}{2}} x + c$
 18. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$ 19. $\frac{3x}{8} + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$
 20. $\frac{1}{16}\sin 2x - \frac{1}{48}\sin^3 2x - \frac{1}{64}\sin 4x - \frac{1}{16}\sin 2x + \frac{1}{16}x + c$ 21. $\frac{3}{2}\sin^{\frac{2}{3}} x - \frac{3}{8}\sin^{\frac{8}{3}} x + c$
 22. $-\frac{1}{\sin x} - 2\sin x + \frac{1}{3}\sin^3 x + c$ 23. $\frac{2}{7}\cos^{\frac{7}{2}} x - \frac{2}{3}\cos^{\frac{3}{2}} x + c$ 24. $\frac{2}{5}\cos^5 \sqrt{x} - \frac{2}{3}\cos^3 \sqrt{x} + c$
 25. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + c$ 26. $-\frac{1}{5}\cot^5 x + \frac{1}{3}\cot^3 x - \cot x - x + c$ 27. $\tan x - x + c$
 28. $\frac{1}{2}\tan^2 x - \ln|\sec x| + c$ 29. $-\frac{1}{2}\cot^2 x - \ln|\sin x| + c$ 30. $-\frac{1}{3}\cot^3 x + \cot x + x + c$
 31. $-\frac{2}{3}\cot 3x - x - \frac{2}{3}\csc 3x + c$ 32. $\frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + c$
 33. $\frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + c$ 34. $-\frac{1}{24}\cot^8 3x - \frac{1}{9}\cot^6 3x - \frac{1}{12}\cot^4 3x + c$ 35. $\frac{2}{7}\tan^{\frac{7}{2}} x + \frac{2}{3}\tan^{\frac{3}{2}} x + c$
 36. $\frac{1}{6}\tan^6 x + \frac{1}{2}\tan^4 x + \frac{1}{2}\tan^2 x + c$ 37. $\csc x + \cot x + c$ 38. $\frac{1}{2}\cos^2 x - \ln|\cos x| + c$
 39. $\frac{1}{2}(\ln|\csc x - \cot x| + \ln|\sec x + \tan x|) + c$ 40. $\frac{1}{2}\sin 2x + c$
 41. $\frac{1}{2\sqrt{2}}\ln\left|\frac{\tan\frac{x}{2}-1-\sqrt{2}}{\tan\frac{x}{2}-1+\sqrt{2}}\right| + \frac{\tan\frac{x}{2}-1}{1+\tan^2\frac{x}{2}} + c$

習題 5-6

1. $\frac{x}{4\sqrt{4-x^2}} + c$ 2. $\frac{8}{3}\sin^{-1}\left(\frac{3x}{4}\right) + \frac{x\sqrt{16-9x^2}}{2} + c$ 3. $\frac{1}{10}\sin^{-1}(5x) + \frac{x\sqrt{1-25x^2}}{2} + c$
 4. $5\sin^{-1}\left(\frac{x-1}{\sqrt{6}}\right) + c$ 5. $\sin^{-1}\left(\frac{x}{3}\right) + c$ 6. $3\ln\left|\frac{3-\sqrt{9-4x^2}}{2x}\right| + \sqrt{9-4x^2} + c$
 7. $\frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{x\sqrt{9-x^2}}{2} + c$ 8. $\frac{1}{4}\tan^{-1}\frac{x}{4} + c$ 9. $\frac{x}{9\sqrt{9+4x^2}} + c$ 10. $\ln\left|\sqrt{x^2+6x+10}+x+3\right| + c$

11. $\frac{x\sqrt{9+x^2}}{2} - \frac{9}{2} \ln|\sqrt{9+x^2}+x| + c$ 12. $\ln|\sqrt{9+x^2}+x| + c$ 13. $\frac{1}{3} \ln\left|\frac{\sqrt{9+4x^2}-3}{2x}\right| + c$
14. $-\frac{\sqrt{4+x^2}}{4x} + c$ 15. $\frac{x}{4\sqrt{4+x^2}} + c$ 16. $\frac{x\sqrt{9+x^2}}{2} + \frac{9}{2} \ln|\sqrt{9+x^2}+x| + c$
17. $\frac{1}{5}(1+x^2)^{\frac{5}{2}} - \frac{1}{3}(1+x^2)^{\frac{3}{2}} + c$ 18. $\sqrt{4x^2-9} - 3\sec^{-1}\frac{2x}{3} + c$ 19. $\ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$
20. $\frac{\sqrt{x^2-1}}{x} + c$ 21. $\frac{25}{54} \sin^{-1}\left(\frac{3x}{5}\right) - \frac{x\sqrt{25-9x^2}}{18} + c$ 22. $\frac{e^x\sqrt{e^{2x}-9}}{2} - \frac{9}{2} \ln(e^x + \sqrt{e^{2x}-9}) + c$
23. $\frac{x\sqrt{x^2-9}}{2} - \frac{9}{2} \ln|x + \sqrt{x^2-9}| + c$ 24. $\frac{1}{2} \sec^{-1}\left(\frac{e^x}{2}\right) + c$ 25. $2x - \frac{7}{2} \tan^{-1}\frac{x+1}{4} + c$
26. $\frac{1}{2} \ln|x^2-2x-8| + \frac{10}{3} \ln\left|\frac{x-4}{\sqrt{x^2-2x-8}}\right| + c$ 27. $\frac{1}{3} \tan^{-1}\frac{x-2}{3} + c$
28. $\ln|x^2-4x+14| + \frac{9\sqrt{10}}{10} \tan^{-1}\frac{x-2}{\sqrt{10}} + c$ 29. $\ln|x-1+\sqrt{x^2-2x}| + c$
30. $-3\sqrt{2x-x^2} + 5\sin^{-1}(x-1) + c$ 31. $-\sqrt{8+2x-x^2} + \sin^{-1}\frac{x-1}{3} + c$
32. $\ln|2x+1+\sqrt{4x^2+4x-16}| + c$ 33. $\sqrt{5-2x+x^2} - 3\ln|\sqrt{x^2-2x+5}+x-1| + c$ 34. $\sqrt{3} - \frac{\pi}{3}$
35. $\sqrt{2} + \ln(\sqrt{2}+1)$ 36. $\frac{25}{6} - \frac{25}{8} \ln 3$

習題 5-7

1. $\frac{1}{4} \ln\left|\frac{x-2}{x+2}\right| + c$ 2. $\frac{3}{2} \ln|x-3| + \frac{1}{2} \ln|x+3| + c$ 3. $\frac{2}{5} \ln|x| - \frac{7}{5} \ln|x+5| + c$
4. $3\ln|x| - \frac{3}{2} \ln(x^2+1) - \tan^{-1}x + c$ 5. $\frac{1}{3}x^3 + 3x + \frac{4}{3} \ln|x-1| - \frac{13}{3} \ln|x+2| + c$
6. $\ln|x| + \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + c$ 7. $\ln|x+1| + \frac{3}{x+1} - \frac{3}{2(x+1)^2} + c$
8. $\frac{3}{25} \ln|x| - \frac{1}{25x} - \frac{3}{50} \ln(x^2+1) - \frac{1}{125} \tan^{-1}x + c$
9. $\frac{5}{12} \ln|x-1| - \frac{3}{4} \ln|x+1| + \frac{3}{2(x+1)} + \frac{4}{3} \ln|x+2| + c$
10. $\frac{1}{2}x^2 - 2x + \frac{1}{8} \ln|x| + \frac{1}{4x} - \frac{1}{4x^2} + \frac{31}{8} \ln|x+2| + c$

$$11. \frac{7}{27} \ln|x+1| + \frac{4}{9(x+1)} + \frac{20}{27} \ln|x-2| - \frac{11}{9(x-2)} + c \quad 12. \frac{1}{32} \ln|x-2| - \frac{1}{32} \ln|x+2| - \frac{1}{16} \tan^{-1} \frac{x}{2} + c$$

$$13. -\ln|\tan x| + \frac{1}{\tan x} + \ln|\tan x - 1| + c \quad 14. x - \ln(e^x + 1) + \frac{1}{e^x + 1} + c$$

$$15. 2\ln|x-1| - 11\ln|x-2| + 10\ln|x-3| + c \quad 16. \ln|x| + \ln|x-1| + \ln|x+2| + c$$

$$17. 2\ln|x-1| - \frac{1}{x-1} - 2\ln|x+2| + c \quad 18. 2\ln|x-1| + \frac{2}{x-1} - \ln(x^2 + 1) + \tan^{-1} x + c$$

$$19. \tan^{-1} x + \frac{1}{2} \ln(x^2 + 4) - \frac{3}{2} \tan^{-1} \frac{x}{2} + c \quad 20. -\frac{1}{x+2} + \frac{3}{2(x+2)^2} + \frac{1}{3(x+2)^3} + c$$

$$21. \ln|x-1| - \frac{4}{x-1} - \frac{3}{(x-1)^2} - \frac{2}{3(x-1)^3} - \frac{1}{2(x-1)^4} + c \quad 22. -\frac{1}{e^x + 1} + \frac{1}{2(e^x + 1)^2} + c$$

$$23. -\frac{1}{9} \ln|e^x + 1| + \frac{1}{3(e^x + 1)} + \frac{1}{9} \ln|e^x - 2| + c \quad 24. -\frac{1}{4} \ln(e^{2x} + 1) - \frac{1}{2} \tan^{-1} e^x + \frac{1}{2} \ln|e^x - 1| + c$$

$$25. \frac{1}{4} \ln|e^x - 1| - \frac{1}{4} \ln|e^x + 3| + c \quad 26. \ln|\sin x| - \frac{1}{2} \ln(\sin^2 x + 1) + c$$

$$27. \sqrt{2} \ln \left| \frac{\sqrt{2} + \tan \frac{x}{2} - 1}{\sqrt{1 + 2 \tan \frac{x}{2} - \tan^2 \frac{x}{2}}} \right| + c \quad 28. -\tan \frac{x}{2} + x + c \quad 29. \frac{2}{5} \ln \left| \frac{2 \tan \frac{x}{2} + 1}{\sqrt{4 - 4 \tan^2 \frac{x}{2} + 6 \tan \frac{x}{2}}} \right| + c$$

$$30. \frac{1}{2} \tan^{-1} \frac{5 \tan \theta + 3}{4} + c \quad 31. \frac{2}{\sqrt{5}} \tan^{-1} \left(\sqrt{5} \tan \frac{x}{2} \right) + c \quad 32. \frac{2}{3} \tan^{-1} \left(\frac{5 \tan \frac{x}{2} + 4}{3} \right) + c$$

$$33. \sqrt{2} \ln \left| \frac{\sqrt{2} + \tan \frac{x}{2} - 1}{\sqrt{1 + 2 \tan \frac{x}{2} - \tan^2 \frac{x}{2}}} \right| + c \quad 34. \ln \left| \frac{\tan \frac{x}{2}}{\tan \frac{x}{2} + 1} \right| + c$$

$$35. -\ln \left| 1 + \tan \frac{x}{2} \right| + \frac{1}{2} \ln \left| 1 + \tan^2 \frac{x}{2} \right| + \frac{x}{2} + c$$

$$36. -\frac{1}{2} \ln \left| 1 - \tan \frac{x}{2} \right| + \frac{1}{2} \ln \left| 1 + \tan \frac{x}{2} \right| - \frac{1}{1 + \tan \frac{x}{2}} + \frac{1}{(1 + \tan \frac{x}{2})^2} + c$$

習題 5-8

1.收斂 2.收斂 3.發散 4.收斂 5.收斂 6.發散 7.收斂 8.發散 9.發散 10.收斂 11.收斂 12.發散
13.收斂 14.收斂 15.發散 16.收斂。

習題 5-9

1. 梯形法則 ≈ 604.06 ，辛普森法則 ≈ 596.25 2. 梯形法則 ≈ 5.6724 ，辛普森法則 ≈ 5.6527
3. 梯形法則 ≈ 0.6938 ，辛普森法則 ≈ 0.6932 4. 梯形法則 ≈ 0.9871 ，辛普森法則 ≈ 1.0001
5. 梯形法則 ≈ 1.3694 ，辛普森法則 ≈ 1.3708 6. 梯形法則 ≈ 0.7462 ，辛普森法則 ≈ 0.7468
7. 梯形法則 ≈ 1.1352 ，辛普森法則 ≈ 1.1358