

## 第 2 章 習題簡答

### 習題 2-1

1. (1)3 (2)1/2 (3)不存在 (4)-1 (5)-1/6 (6) $e^2$

2. (1) $\frac{-1}{\sqrt{3-2x}}$  (2) $\frac{-1}{\sqrt{3}}$  (3) $\frac{-1}{\sqrt{5}}$

3.  $f(a)-af'(a)$  4. 略 5.  $y-1=-2(x+1)$

### 習題 2-2

1. (1)  $f'(x)=9x^8-7$  (2)  $f'(x)=1-\frac{1}{(x-1)^2}$  (3)  $f'(x)=\frac{-1}{x^2}-\frac{2}{x^3}$  (4)  $f'(x)=1-4x$

(5)  $g'(x)=2(3x^2-5x)(6x-5)$  (6)  $g'(x)=(3x^2+2)(x^4-5x)+(x^3+2x+1)(4x^3-5)$

(7)  $g'(x)=2x(x^2+3)(x^2+5)+(x^2+1)2x(x^2+5)+(x^2+1)(x^2+3)2x$

(8)  $g'(x)=\frac{-30x}{(3x^2+2)^2}$  (9)  $h'(x)=\frac{-25}{(3x-4)^2}$  (10)  $h'(x)=\frac{-(4x^2+8x-3)}{(4x^2-3x)^2}$

(11)  $h'(x)=-6x^{-3}+(4x^3-7)(3x+5)+(x^4-7x)3$

(12)  $\frac{(7x-6)[(3x^2-4)(x^2+x)+(x^3+4x-9)(2x+1)]-(x^3+4x-9)(x^2+x)7}{(7x-6)^2}$

2.  $f'(1)$  不存在

$$3. f'(x) = \begin{cases} 2x+1, & x > 0 \\ 2x-1, & x < 0 \\ \text{不存在}, & x = 0 \end{cases}$$

4.  $y-2=-\frac{3}{2}(x-1)$

### 習題 2-3

1. (1)  $f'(x)=20(x-1)(x^2-2x+5)^9$  (2)  $y'=3x\sqrt{x^2-5}$

(3)  $g'(x)=(x^3+x-1)(x^2+3)^2[2(3x^2+1)(x^2+3)+6x(x^3+x-1)^2]$  (4)  $y'=\frac{96x^2}{(5-4x^3)^5}$

(5)  $h'(s)=-\frac{6(2s+3)^2(s^2+3s+5)}{(s^2-5)^4}$  (6)  $y'=\frac{-w^2+8w+4}{(4-3w^2)^{2/3}}$  (7)  $l'(x)=\frac{5}{3\sqrt[3]{x^2}(1-\sqrt[3]{x})^6}$

(8)  $y'=-\frac{5}{t^2}\left(1+\frac{1}{t}\right)^4$  (9)  $k'(x)=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)$

(10)  $y'=-3x^{-4}+\frac{1}{5}(x^3+2x)^{-4/5}(3x^2+2)$

$$2. -475 \quad 3. w'(x) = \frac{-3(3z^2 + 2)}{x^4} \quad 4. \frac{dy}{dx} = -\frac{1}{2\sqrt[3]{x^2}} \left(6u + \frac{1}{2\sqrt{u}}\right)(24v^2 + 5)$$

$$5. f'(2) = 12 \quad 6. f'(27) = \frac{2}{9} \quad 7. D_x f(2x+3) = 2g(2x+3)$$

$$8. y - \sqrt{10} = \frac{13}{2\sqrt{10}}(x-2)$$

### 習題 2-4

$$1. (1) -\frac{x^2}{y^2} \quad (2) \frac{1-y^2-2xy}{2xy+x^2+1} \quad (3) -\frac{\sqrt[3]{y}}{\sqrt[3]{x}} \quad (4) \frac{4x\sqrt{xy}+y}{4y\sqrt{xy}-x}$$

$$2. \frac{dy}{dx} = \frac{2xy^2-3x^2-y}{x-4y^3-2x^2y}, \quad \frac{dx}{dy} = \frac{x-4y^3-2x^2y}{2xy^2-3x^2-y} \quad 3. \frac{1}{4y^3-6y} \quad 4. 0$$

$$5. f(1) = 3, f^{-1}(3) = 1, f'(1) = 6, (f^{-1})'(3) = \frac{1}{6} \quad 6. \frac{1}{2}$$

$$7. \text{切線斜率: } \frac{13}{9}, \text{ 切線方程式: } y-1 = \frac{13}{9}(x-3)$$

$$8. y-5 = \frac{5}{2}(x-2) \quad 9. y - \frac{2}{\sqrt{3}} = 2\left(x - \frac{4}{\sqrt{3}}\right)$$

### 習題 2-5

$$1. (1) e^{\frac{1}{2}} \quad (2) e^r \quad (3) e^{\frac{1}{3}} \quad (4) e$$

$$2. (1) 1 + \ln x \quad (2) \frac{3}{x} \quad (3) \frac{5(\ln x)^4}{x} \quad (4) \frac{2x}{1+x^2} \quad (5) \frac{1}{1+x} + \frac{1}{1-x} \quad (6) \frac{1}{x} - \frac{x^2}{1+x^3}$$

$$(7) \frac{3}{(3x+1)\ln 2} \quad (8) \frac{(2x+1)\ln x - (x+1)\ln(x^2+x)}{(x^2+x)(\ln x)^2} \quad (9) \frac{-(y^3+2x^2y\ln y)}{x^3+2xy\ln x}$$

$$(10) -\frac{2x+y(x^2+y^2)\ln 3}{2y+x(x^2+y^2)\ln 3}$$

$$3. (1) (1+x)e^x \quad (2) 2e^{2x} \quad (3) e^x \left( \ln x + \frac{1}{x} \right) \quad (4) 2xe^{x^2} \quad (5) 3^x \ln 3 + 3x^2$$

$$(6) (2x+1)5^{x^2+x} \ln 5 \quad (7) \frac{4}{(e^x + e^{-x})^2} \quad (8) \frac{xe^{(x+1)(x+2)}}{(x+1)^2(x+2)^3} \left[ \frac{1}{x} + 2x + 3 - \frac{2}{x+1} - \frac{3}{x+2} \right]$$

$$(9) x^{x^2} (2x \ln x + x) \quad (10) \frac{y^x \ln y - y \cdot x^{y-1}}{x^y \ln x - x \cdot y^{x-1}}$$

$$4. \pi(1 + \ln \pi) \quad 5. f(e) = 1, f'(e) = \frac{1}{e} \quad 6. y - e = 3e(x-1)$$

$$7. \text{切線斜率: } 2, \text{ 切線方程式: } y = 2x + 1.$$

習題 2-6

1. (1)  $\frac{1}{4}$  (2)  $\frac{2}{5}$  (3)  $\frac{1}{2}$  (4)  $\frac{1}{2}$

2. (1)  $3\cos x + 4\sin x$  (2)  $3\sin^2 x \cdot \cos x$  (3)  $\frac{-2 + \cos x}{(1 - 2\cos x)^2}$

(4)  $\frac{1}{\sqrt{x^3}} \csc^2 \sec^2 \frac{1}{\sqrt{x}} \cdot \sec^2 \frac{1}{\sqrt{x}} \cdot \tan \frac{1}{\sqrt{x}}$  (5)  $2x \sec x + x^2 \sec x \tan x$

(6)  $12 \tan^3 3x \sec^2 3x$  (7)  $\csc x$  (8)  $e^{\sin x} (1 + x \cos x)$

(9)  $\frac{-(2x+1)}{3\sqrt{(x^2+x+1)^2}} \cdot \sin(2\sqrt{x^2+x+1})$  (10)  $\frac{6\sin^2 x \cos x(1-3\cos x) - 3\sin^4 x}{2\sqrt{(1-3\cos x)^3}}$  (11) 0

(12)  $\frac{-1}{2\sqrt{x}\sqrt{1-x}}$  (13)  $\frac{1-2x^2}{\sqrt{1-x^2+x^4}\sqrt{1-x^2}}$  (14)  $\frac{x}{|x|(x^2+1)}$  (15)  $\frac{2}{1+x^2}$

(16)  $\frac{1}{1+x^2}$  (17)  $\frac{-1}{\sqrt{e^{2x}-1}}$  (18)  $2x \sec^{-1} \sqrt{1+x^2} + \frac{x^3}{|x|(1+x^2)}$  (19)  $4x \cosh(2x^2+1)$

(20)  $20x \tanh(5x^2) \sec h^2(5x^2)$  (21)  $\sinh x \cdot e^{\cosh x}$  (22)  $\coth(3x) - 3x \csc h^2(3x)$

(23)  $\coth^{-1}(3x) + \frac{3x}{1-9x^2}$  (24)  $\frac{1}{\sqrt{a^2+x^2}}$  (25)  $\sec x$

(26)  $(\sinh x)^{\sin x} (\cos x \cdot \ln(\sinh x) + \sin x \cdot \coth x)$

3. (1)  $\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$  (2)  $\frac{dy}{dx} = \frac{2x \cos y + \cos(x-y)}{x^2 \sin y + \cos(x-y)}$  (3)  $\frac{dy}{dx} = -\frac{y}{x}$

(4)  $\frac{dy}{dx} = \frac{y - 2 \tan(x+y) \sec^2(x+y)}{2 \tan(x+y) \sec^2(x+y) - x + \csc^2 y}$

(5)  $\frac{dy}{dx} = \frac{\csc(x-y) \cot(x-y) - \sec(x+y) \tan(x+y) + 1}{\csc(x-y) \cot(x-y) + \sec(x+y) \tan(x+y)}$

(6)  $\frac{dy}{dx} = \frac{y^{\sin x} \cos x \cdot \ln y - x^{y-1} \cdot y}{x^y \cdot \ln x - \sin x \cdot y^{\sin x - 1}}$  (7)  $\frac{dy}{dx} = \frac{y\sqrt{1-(x+y)^2} + \sqrt{1-(xy)^2}}{-\sqrt{1-(xy)^2} - x\sqrt{1-(x+y)^2}}$

(8)  $\frac{dy}{dx} = \frac{(1+y^2)(\sin y + 2x)}{1-x(1+y^2)\cos y}$  (9)  $\frac{dy}{dx} = \frac{2xy + y \cosh(x+y^2)}{1-2y^2 \cosh(x+y^2)}$

(10)  $\frac{dy}{dx} = \frac{1}{1 + \sqrt{(x-y)^2 + 1}}$

4. 略 5. 略 6. 略 7. 略

8. 切線斜率 =  $-\frac{1}{2\sqrt{3}}$ ，切線方程式： $y - (1 + \frac{\sqrt{3}}{2}) = -\frac{1}{2\sqrt{3}}(x-1)$

9. 切線斜率 =  $-\frac{1}{2}$ ，切線方程式： $y = -\frac{1}{2}x + \pi$

習題 2-7

1. (1)  $10x^9 - 35x^6$  (2)  $90x^8 - 210x^5$  (3)  $10!$  2.  $f''(2) = 8$  3.  $y^{(n)} = \frac{3^n n!}{(1-3x)^{n+1}}$

4.  $y^{(n)} = -\frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n} \cdot \frac{1}{(1-x)^{\frac{2n-1}{2}}}$  5.  $y^{(n)} = (-1)^n \frac{n!}{3} \left[ \frac{4}{(x+1)^{n+1}} - \frac{1}{(x-2)^{n+1}} \right]$

6.  $f'(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$ ,  $f''(x) = \begin{cases} 2, & x > 0 \\ \text{不存在}, & x = 0 \\ -2, & x < 0 \end{cases}$  7.  $-\frac{x^2 + y^2}{y^3}$

8.  $f^{(n)}(x) = (-1)^{n-1} (n-1)! \frac{1}{(1+x)^n}$ ,  $f^{(2008)}(0) = -2007!$

9.  $f^{(n)}(x) = \begin{cases} \sin x, & n = 4k \\ \cos x, & n = 4k + 1 \\ -\sin x, & n = 4k + 2 \\ -\cos x, & n = 4k + 3 \end{cases} \quad k = 0, 1, 2, \dots$  10.  $f^{(n)}(x) = a^n e^{ax}$  11. 略。