第 10 章 習題簡答

習題 10-1

1. $\sum_{n=1}^{\infty}\frac{7}{(n+3)(n+4)}=\frac{7}{4}$，收斂 2.發散 3.發散 4. $\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n=4$，收斂 5.發散 6.發散

7. $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{4^{n+1}}=-\frac{3}{28}$，收斂

習題 10-2

1.收斂 2.發散 3.收斂 4.收斂 5.收斂 6.收斂 7.收斂 8.發散

習題 10-3

1. 收斂 2.收斂 3.收斂

習題 10-4

1.(1) $f(x)=\frac{1}{2+x}=\frac{1}{2}(1-\frac{x}{2}+\frac{x^2}{2^2}-\frac{x^3}{2^3}+\cdots+(-1)^n\frac{x^n}{2^n}+\cdots),\forall|x|<2$

(2) $f(x)=\frac{x^3}{1-x}=x^3+x^4+\cdots+x^n+\cdots,\forall|x|<1$

(3) $f(x)=\frac{1}{2+5x}=\frac{1}{2}(1-\frac{5}{2}x+\frac{5^2}{2^2}x^2-\frac{5^3}{2^3}x^3+\cdots+(-1)^n\frac{5^n}{2^n}x^n+\cdots),\forall|x|<\frac{2}{5}$

2.(1) $(-\infty,\infty)$ (2) $(-\frac{5}{2},\frac{1}{2})$ (3) $[-\frac{1}{2},\frac{1}{2})$ 3. 0.3333

4.(1) $f(x)=\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n}}{2n\cdot n!},-\infty<x<\infty$ (2) $f(x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+3}}{(n+1)(2n+3)},|x|<1$

習題 10-5

1.(1) $f(x)=\frac{1}{\sqrt{2}}[1+(x-\frac{\pi}{4})-\frac{1}{2!}(x-\frac{\pi}{4})^2-\frac{1}{3!}(x-\frac{\pi}{4})^3+\cdots]$

(2) $f(x)=\frac{1}{2}-\frac{\sqrt{3}}{2}(x-\frac{\pi}{3})-\frac{1}{2\cdot 2!}(x-\frac{\pi}{3})^2+\frac{\sqrt{3}}{2}\cdot\frac{1}{3!}(x-\frac{\pi}{3})^3+\frac{1}{2\cdot 4!}(x-\frac{\pi}{3})^4+\cdots$

(3) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}$ (4) $f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}x^{n+1}$

2.(1) $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n2^n}{n!}x^{n+1}$ (2) $f(x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+3}}{(2n+1)!}$ (3) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n)!}x^{2n-1}$

3.(1) 0.1248 (2) 0.7966 (3) 0.4485