命題老師簽名： 陳福坤 **(以 B4 列印)**	考試時間：97 年 06 月 12 日(星期四)第 1 節	答案紙：□要附■不附□自訂
南台科技大學 96 學年度第 二 學期□期中□期末考**試題**	開課班級：四技資工二甲、四技資工二乙	※試題另附答案紙者請一併收回
科目： 線性代數	班級： 技 系 年 班 學號： 姓名：	可攜帶物品：計算機

※「考試作弊會受到大過以上、成績零分計算之懲處」※

1. <Sol.>

Since the normalized basis is $\mathbf{u}_1 = (1,0,0)$, $\mathbf{u}_2 = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, we have

$$\text{proj}_W \mathbf{v} = (\mathbf{v}\cdot\mathbf{u}_1)\mathbf{u}_1 + (\mathbf{v}\cdot\mathbf{u}_2)\mathbf{u}_2 = [(4,1,-7)\cdot(1,0,0)](1,0,0) + \left[(4,1,-7)\cdot(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\right](0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = (4,0,0)+(0,-3,-3) = (4,-3,-3)$$

Thus, $d(\mathbf{x},W) = \min\{d(\mathbf{x},\mathbf{y})\} = \|\mathbf{x} - \text{proj}_W \mathbf{x}\| = \|(4,1,-7)-(4,-3,-3)\| = \|(0\,4,-4)\| = \sqrt{32}$

2. <Sol.> Two polynomials $\mathbf{u}_1 = a_1x^2 + b_1x + c_1$, $\mathbf{u}_2 = a_2x^2 + b_2x + c_2$

$$T(a_1x^2 + b_1x + c_1) = (a_1 + b_1)x + c_1$$
$$T(a_2x^2 + b_2x + c_2) = (a_2 + b_2)x + c_2$$
$$T(d_1\mathbf{u}_1 + d_2\mathbf{u}_2) = T(d_1[a_1x^2 + b_1x + c_1] + d_2[a_2x^2 + b_2x + c_2]) = T([d_1a_1 + d_2a_2]x^2 + [d_1b_1 + d_2b_2]x + [d_1c_1 + d_2c_2])$$
$$= [d_1a_1 + d_2a_2 + d_1b_1 + d_2b_2]x + [d_1c_1 + d_2c_2] = [(d_1a_1 + d_1b_1)x + d_1c_1] + [(d_2a_2 + d_2b_2)x + d_2c_2] = d_1T(\mathbf{u}_1) + d_2T(\mathbf{u}_2)$$
$$= d_1[(a_1 + b_1)x + c_1] + d_2[(a_2 + b_2)x + c_2] = d_1T(a_1x^2 + b_1x + c_1) + d_2T(a_2x^2 + b_2x + c_2) = d_1T(\mathbf{u}_1) + d_2T(\mathbf{u}_2)$$

3. <Sol.>

(a) Eigen-structure: $\lambda_1 = 0$, $\mathbf{v}_1 = r\begin{bmatrix}1\\-1\\1\end{bmatrix}$, $\lambda_2 = 3$, $\mathbf{v}_2 = s\begin{bmatrix}1\\1\\0\end{bmatrix} + t\begin{bmatrix}-1\\0\\1\end{bmatrix}$, $C = \begin{bmatrix}1 & 1 & -1\\-1 & 1 & 0\\1 & 0 & 1\end{bmatrix}$

(b) $D = \begin{bmatrix}0 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$

(c) $A^{100} = CD^{100}C^{-1} = \begin{bmatrix}1 & 1 & -1\\-1 & 1 & 0\\1 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & -1\\-1 & 1 & 0\\1 & 0 & 1\end{bmatrix}^{-1}$

(d) Applying Gram-Schmidt process,

$\mathbf{v}_1 = (1,-1,1)$, $\mathbf{v}_2 = (1,1,0)$, $\mathbf{v}_3 = (-1,0,1)$

$\mathbf{u}_1 = \mathbf{v}_1 = (1,-1,1)$

$$\mathbf{u}_2 = \mathbf{v}_2 - \text{proj}_{\mathbf{u}_1}\mathbf{v}_2 = \mathbf{v}_2 - \frac{\mathbf{v}_2\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 = (1,1,0) - \frac{0}{3}(1,-1,1) = (1,1,0)$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \text{proj}_{\mathbf{u}_1}\mathbf{v}_3 - \text{proj}_{\mathbf{u}_2}\mathbf{v}_3 = \mathbf{v}_3 - \frac{\mathbf{v}_3\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 - \frac{\mathbf{v}_3\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 = (-1,0,1) - \frac{0}{3}(1,-1,1) - \frac{0}{2}(1,1,0) = (-1,0,1)$$

$\|(1,-1,1)\| = \sqrt{3}$, $\|(1,1,0)\| = \sqrt{2}$, $\|(-1,0,1)\| = \sqrt{2}$

The orthonormal basis for V is $\left\{(\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0),(\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}})\right\}$