## EXERCISES 3.3, page 195

2. $f(x)=(1-x)^3$; $f'(x)=3(1-x)^2(-1)=-3(1-x)^2$.

3. $f(x)=(x^2+2)^5$. $f'(x)=5(x^2+2)^4(2x)=10x(x^2+2)^4$.

6. $f(x)=3(x^3-x)^4$. $f'(x)=(3)(4)(x^3-x)^3(3x^2-1)=12(3x^2-1)(x^3-x)^3$.

8. $f(t)=\frac{1}{2}(2t^2+t)^{-3}$. $f'(t)=\frac{1}{2}(-3)(2t^2+t)^{-4}(4t+1)=-\dfrac{3(1+4t)}{2(2t^2+t)^4}$.

10. $f(t)=(3t^2-2t+1)^{3/2}$. $f'(t)=\frac{3}{2}(3t^2-2t+1)^{1/2}(6t-2)=3(3t-1)(3t^2-2t+1)^{1/2}$

11. $f(x)=\sqrt{3x-2}=(3x-2)^{1/2}$.

$$f'(x)=\frac{1}{2}(3x-2)^{-1/2}(3)=\frac{3}{2}(3x-2)^{-1/2}=\frac{3}{2\sqrt{3x-2}}.$$

14. $f(x)=\sqrt{2x^2-2x+3}$.

$$f'(x)=\tfrac{1}{2}(2x^2-2x+3)^{-1/2}(4x-2)=(2x-1)(2x^2-2x+3)^{-1/2}.$$

16. $f(x)=\dfrac{2}{(x^2-1)^4}$.

$$f'(x)=2\frac{d}{dx}(x^2-1)^{-4}=2(-4)(x^2-1)^{-5}(2x)=-16x(x^2-1)^{-5}.$$

18. $f(x)=\dfrac{1}{\sqrt{2x^2-1}}=(2x^2-1)^{-1/2}$. $f'(x)=-\dfrac{1}{2}(2x^2-1)^{-3/2}(4x)=-\dfrac{2x}{\sqrt{(2x^2-1)^3}}$.

19. $y=\dfrac{1}{(4x^4+x)^{3/2}}$.

$$\frac{dy}{dx}=\frac{d}{dx}(4x^4+x)^{-3/2}=-\frac{3}{2}(4x^4+x)^{-5/2}(16x^3+1)=-\frac{3}{2}(16x^3+1)(4x^4+x)^{-5/2}.$$

24. $f(t)=(2t-1)^4+(2t+1)^4$.

$$f'(t)=4(2t-1)^3(2)+4(2t+1)^3(2)=8[(2t-1)^3+(2t+1)^3].$$

26. $f(v)=(v^{-3}+4v^{-2})^3$. $f'(v)=3(v^{-3}+4v^{-2})^2(-3v^{-4}-8v^{-3})$.

27. $f(x)=\sqrt{x+1}+\sqrt{x-1}=(x+1)^{1/2}+(x-1)^{1/2}$.

$f'(x)=\frac{1}{2}(x+1)^{-1/2}(1)+\frac{1}{2}(x-1)^{-1/2}(1)=\frac{1}{2}[(x+1)^{-1/2}+(x-1)^{-1/2}]$.

29. $f(x)=2x^2(3-4x)^4$.

$$f'(x)=2x^2(4)(3-4x)^3(-4)+(3-4x)^4(4x)=4x(3-4x)^3(-8x+3-4x)$$
$$=4x(3-4x)^3(-12x+3)=(-12x)(4x-1)(3-4x)^3.$$

31. $f(x)=(x-1)^2(2x+1)^4$.

$$f'(x)=(x-1)^2\frac{d}{dx}(2x+1)^4+(2x+1)^4\frac{d}{dx}(x-1)^2 \quad \text{[Product Rule]}$$
$$=(x-1)^2(4)(2x+1)^3\frac{d}{dx}(2x+1)+(2x+1)^4(2)(x-1)\frac{d}{dx}(x-1)$$
$$=8(x-1)^2(2x+1)^3+2(x-1)(2x+1)^4$$
$$=2(x-1)(2x+1)^3(4x-4+2x+1)=6(x-1)(2x-1)(2x+1)^3.$$

32. $g(u)=(1+u^2)^5(1-2u^2)^8$.

$$g'(u)=(1+u^2)^5(8)(1-2u^2)^7(-4u)+(1-2u^2)^8(5)(1+u^2)^4(2u)$$
$$=-2u(1+u^2)^4(1-2u^2)^7[16(1+u^2)-5(1-2u^2)]$$
$$=-2u(26u^2+11)(1+u^2)^4(1-2u^2)^7.$$

33. $f(x)=\left(\frac{x+3}{x-2}\right)^3$.

$$f'(x)=3\left(\frac{x+3}{x-2}\right)^2\frac{d}{dx}\left(\frac{x-3}{x-2}\right)=3\left(\frac{x+3}{x-2}\right)^2\left[\frac{(x-2)(1)-(x+3)(1)}{(x-2)^2}\right]$$

$$=3\left(\frac{x+3}{x-2}\right)^2\left[-\frac{5}{(x-2)^2}\right]=-\frac{15(x+3)^2}{(x-2)^4}.$$

38. $g(x)=\left(\frac{2x+1}{2x-1}\right)^{1/2}$.

$$g'(x)=\frac{1}{2}\left(\frac{2x+1}{2x-1}\right)^{-1/2}\left[\frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{2x-1}\right)^{-1/2}\left(-\frac{4}{(2x-1)^2}\right)=-\frac{2}{(2x+1)^{1/2}(2x-1)^{3/2}}.$$

39. $f(x)=\dfrac{x^2}{(x^2-1)^4}$.

$$f'(x)=\frac{(x^2-1)^4\dfrac{d}{dx}(x^2)-(x^2)\dfrac{d}{dx}(x^2-1)^4}{\left[(x^2-1)^4\right]^2}$$
$$=\frac{(x^2-1)^4(2x)-x^2(4)(x^2-1)^3(2x)}{(x^2-1)^8}$$
$$=\frac{(x^2-1)^3(2x)(x^2-1-4x^2)}{(x^2-1)^8}=\frac{(-2x)(3x^2+1)}{(x^2-1)^5}.$$

42. $g(t)=\dfrac{(2t-1)^2}{(3t+2)^4}$.

$$g'(t)=\frac{(3t+2)^4(2)(2t-1)(2)-(2t-1)^2(4)(3t+2)^3(3)}{(3t+2)^8}$$
$$=\frac{2(3t+2)^3(2t-1)[2(3t+2)-6(2t-1)]}{(3t+2)^8}=\frac{4(2t-1)(5-3t)}{(3t+2)^5}.$$

43. $f(x)=\dfrac{\sqrt{2x+1}}{x^2-1}$.

$$f'(x)=\frac{(x^2-1)(\tfrac{1}{2})(2x+1)^{-1/2}(2)-(2x+1)^{1/2}(2x)}{(x^2-1)^2}$$
$$=\frac{(2x+1)^{-1/2}[(x^2-1)-(2x+1)(2x)]}{(x^2-1)^2}=-\frac{3x^2+2x+1}{\sqrt{2x+1}(x^2-1)^2}.$$

45. $g(t)=\dfrac{(t+1)^{1/2}}{(t^2+1)^{1/2}}$.

$$g'(t)=\frac{(t^2+1)^{1/2}\dfrac{d}{dt}(t+1)^{1/2}-(t+1)^{1/2}\dfrac{d}{dt}(t^2+1)^{1/2}}{t^2+1}$$
$$=\frac{(t^2+1)^{1/2}(\tfrac{1}{2})(t+1)^{-1/2}(1)-(t+1)^{1/2}(\tfrac{1}{2})(t^2+1)^{-1/2}(2t)}{t^2+1}$$
$$=\frac{\tfrac{1}{2}(t+1)^{-1/2}(t^2+1)^{-1/2}[(t^2+1)-2t(t+1)]}{t^2+1}=-\frac{t^2+2t-1}{2\sqrt{t+1}(t^2+1)^{3/2}}.$$

47. $f(x) = (3x+1)^4(x^2-x+1)^3$

$$f'(x) = (3x+1)^4 \cdot \frac{d}{dx}(x^2-x+1)^3 + (x^2-x+1)^3 \frac{d}{dx}(3x+1)^4$$
$$= (3x+1)^4 \cdot 3(x^2-x+1)^2(2x-1) + (x^2-x+1)^3 \cdot 4(3x+1)^3 \cdot 3$$
$$= 3(3x+1)^3(x^2-x+1)^2[(3x+1)(2x-1) + 4(x^2-x+1)]$$
$$= 3(3x+1)^3(x^2-x+1)^2(6x^2-3x+2x-1+4x^2-4x+4)$$
$$= 3(3x+1)^3(x^2-x+1)^2(10x^2-5x+3)$$

49. $y = g(u) = u^{4/3}$ and $\frac{dy}{du} = \frac{4}{3}u^{1/3}$, $u = f(x) = 3x^2-1$, and $\frac{du}{dx} = 6x$.

So $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{4}{3}u^{1/3}(6x) = \frac{4}{3}(3x^2-1)^{1/3}6x = 8x(3x^2-1)^{1/3}$.

53. $\frac{dy}{du} = \frac{1}{2}u^{-1/2} - \frac{1}{2}u^{-3/2}$, $\frac{du}{dx} = 3x^2-1$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left[\frac{1}{2\sqrt{x^3-x}} - \frac{1}{2(x^3-x)^{3/2}}\right](3x^2-1)$$
$$= \frac{(3x^2-1)(x^3-x-1)}{2(x^3-x)^{3/2}}.$$

55. $F(x) = g(f(x))$; $F'(x) = g'(f(x))f'(x)$ and $F'(2) = g'(3)(-3) = (4)(-3) = -12$

57. Let $g(x) = x^2+1$, then $F(x) = f(g(x))$. Next, $F'(x) = f'(g(x))g'(x)$ and $F'(1) = f'(2)(2x) = (3)(2) = 6$.

63. $f(x) = x\sqrt{2x^2+7}$. $f'(x) = \sqrt{2x^2+7} + x(\frac{1}{2})(2x^2+7)^{-1/2}(4x)$.

The slope of the tangent line is $f'(3) = \sqrt{25} + (\frac{3}{2})(25)^{-1/2}(12) = \frac{43}{5}$.

An equation of the tangent line is $y - 15 = \frac{43}{5}(x-3)$ or $y = \frac{43}{5}x - \frac{54}{5}$.