EX 1.4

19. The slope of the line through $A$ and $B$ is $\frac{-10-(-2)}{-3-1}=\frac{-8}{-4}=2$.

The slope of the line through $C$ and $D$ is $\frac{1-5}{-1-1}=\frac{-4}{-2}=2$.

Since the slopes of these two lines are equal, the lines are parallel.

21. The slope of the line through $A$ and $B$ is $\frac{2-5}{4-(-2)}=-\frac{3}{6}=-\frac{1}{2}$.

The slope of the line through $C$ and $D$ is $\frac{6-(-2)}{3-(-1)}=\frac{8}{4}=2$. Since the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

27. We use the point-slope form of an equation of a line with the point (3,– 4) and slope $m=2$. Thus $y-y_1=m(x-x_1)$,

and

$$y-(-4)=2(x-3)$$
$$y+4=2x-6$$
$$y=2x-10.$$

31. We first compute the slope of the line joining the points (2,4) and (3,7). Thus,

$$m=\frac{7-4}{3-2}=3.$$

Using the point-slope form of an equation of a line with the point (2,4) and slope $m=3$, we find

$$y-4=3(x-2)$$
$$y=3x-2.$$

35. We use the slope-intercept form of an equation of a line: $y=mx+b$. Since $m=3$, and $b=4$, the equation is $y=3x+4$.

41. We write the equation in slope-intercept form:

$$2x-3y-9=0$$
$$-3y=-2x+9$$
$$y=\tfrac{2}{3}x-3.$$

From this equation, we see that $m=2/3$ and $b=-3$.

45. We first write the equation $2x - 4y - 8 = 0$ in slope- intercept form:

$$2x-4y-8=0$$
$$4y=2x-8$$
$$y=\tfrac{1}{2}x-2$$

Now the required line is parallel to this line, and hence has the same slope. Using the point-slope equation of a line with $m = 1/2$ and the point $(-2,2)$, we have

$$y-2=\tfrac{1}{2}[x-(-2)]$$
$$y=\tfrac{1}{2}x+3.$$

46. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form:

$$3x+4y-22=0$$
$$4y=-3x+22$$
$$y=-\tfrac{3}{4}x+\tfrac{22}{4}.$$

Now the required line is perpendicular to this line, and hence has slope 4/3 (the negative reciprocal of $-3/4$). Using the point-slope equation of a line with $m = 4/3$ and the point $(2,4)$, we have

$$y-4=\tfrac{4}{3}(x-2)$$
$$y=\tfrac{4}{3}x+\tfrac{4}{3}$$

51. Since the required line is parallel to the line joining $(-3,2)$ and $(6,8)$, it has slope

$$m=\frac{8-2}{6-(-3)}=\frac{6}{9}=\frac{2}{3}.$$

We also know that the required line passes through $(-5,-4)$. Using the point-slope form of an equation of a line, we find

$$y-(-4)=\frac{2}{3}(x-(-5))$$

or $\quad y=\tfrac{2}{3}x+\tfrac{10}{3}-4;\;$ that is $\quad y=\tfrac{2}{3}x-\tfrac{2}{3}$

53. Since the point $(-3,5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives

$$-3k+15+9=0,\quad \text{or} \quad k=8.$$

63. Using the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ with $a = -2$ and $b = -4$, we have $-\dfrac{x}{2}-\dfrac{y}{4}=1$.

Then

$$-4x-2y=8$$
$$2y=-8-4x$$
$$y=-2x-4.$$

83. Writing each equation in the slope-intercept form, we have

$$y=-\frac{a_1}{b_1}x-\frac{c_1}{b_1}\quad (b_1\neq 0) \quad \text{and} \quad y=-\frac{a_2}{b_2}x-\frac{c_2}{b_2}\quad (b_2\neq 0)$$

Since two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $-\dfrac{a_1}{b_1}=-\dfrac{a_2}{b_2}$, or $a_1b_2 - b_1a_2 = 0$.