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1. $\lim_{x\to 2^-} f(x) = 3$, $\lim_{x\to 2^+} f(x) = 2$, $\lim_{x\to 2} f(x)$ does not exist.

3. $\lim_{x\to -1^-} f(x) = \infty$, $\lim_{x\to -1^+} f(x) = 2$. Therefore $\lim_{x\to -1} f(x)$ does not exist.

25. $\lim_{x\to 0^+} \frac{1}{x}$ does not exist because $1/x \to \infty$ as $x \to 0$ from the right.

30. $\lim_{x\to 2^+} 2\sqrt{x-2} = 2\cdot 0 = 0.$

34. $\lim_{x\to 1^+} \frac{1+x}{1-x} = -\infty.$

40. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(2x+3) = 3$, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-x+1) = 1.$

51. f is continuous for all values of x.

53. f is continuous for all values of x. Note that $x^2 + 1 \geq 1 > 0$.

60. Observe that $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-}(-x+1) = 2 \neq \lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}(x+1) = 0$, and so f is discontinuous at $x = -1$.

66. f is not defined at $x = 1$ and is discontinuous there. It is continuous everywhere else.

83. We require that $f(1) = 1 + 2 = 3 = \lim_{x\to 1^+} kx^2 = k$, or $k = 3$.

84. Since $\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)(x+2)}{x+2} = \lim_{x\to -2}(x-2) = -4$, we define $f(-2) = k = -4$, that is, take $k = -4$.

97. False. Take

$$f(x)=\begin{cases}-1 & \text{if } x<2\\ 4 & \text{if } x=2\\ 1 & \text{if } x>2\end{cases}$$

Then $f(2)=4$ but $\lim_{x\to 2}$ does not exist.

98. False. Take $f(x)=\begin{cases}x+3 & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$. Then $\lim_{x\to 0} f(x)=3$, but $f(0)=1$.

99. False. Consider the function $f(x)=x^2-1$ on the interval [-2, 2]. Here, $f(-2)=f(2)=3$, but f has zeros at $x=-1$ and $x=1$.

103. False. Take $f(x)=\begin{cases}\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$. Then f is continuous for all $x\neq 0$ but $\lim_{x\to 0} f(x)$ does not exist.

105. True. Since the number 2 lies between $f(-2)=3$ and $f(3)=1$, the intermediate value theorem guarantees that there exists at least one number $-2\le c\le 3$ such that $f(c)=2$.

107. a. Both $g(x)=x$ and $h(x)=\sqrt{1-x^2}$ are continuous on [-1,1] and so $f(x)=x-\sqrt{1-x^2}$ is continuous on [-1,1].

b. $f(-1)=-1$ and $f(1)=1$ and so f has at least one zero in (-1,1).

c. Solving $f(x)=0$, we have $x=\sqrt{1-x^2}$, $x^2=1-x^2$, $2x^2=1$, or $x=\frac{\pm\sqrt{2}}{2}$.