3. a. The actual cost incurred in the production of the 1001st record is given by

$$C(1001) - C(1000) = [2000 + 2(1001) - 0.0001(1001)^2] - [2000 + 2(1000) - 0.0001(1000)^2] = 3901.7999 - 3900 = 1.7999,$$

or \$1.80. The actual cost incurred in the production of the 2001st record is given by

$$C(2001) - C(2000) = [2000 + 2(2001) - 0.0001(2001)^2] - [2000 + 2(2000) - 0.0001(2000)^2] = 5601.5999 - 5600 = 1.5999, \text{ or } \$1.60.$$

b. The marginal cost is $C'(x) = 2 - 0.0002x$. In particular

$$C'(1000) = 2 - 0.0002(1000) = 1.80$$

and

$$C'(2000) = 2 - 0.0002(2000) = 1.60.$$

5. a. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{100x + 200{,}000}{x} = 100 + \dfrac{200{,}000}{x}.$

b. $\overline{C}'(x) = \dfrac{d}{dx}(100) + \dfrac{d}{dx}(200{,}000x^{-1}) = -200{,}000x^{-2} = -\dfrac{200{,}000}{x^2}.$

c. $\lim\limits_{x\to\infty} \overline{C}(x) = \lim\limits_{x\to\infty}\left[100 + \dfrac{200{,}000}{x}\right] = 100$

and this says that the average cost approaches \$100 per unit if the production lev is very high.

7. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{2000 + 2x - 0.0001x^2}{x} = \dfrac{2000}{x} + 2 - 0.0001x.$

$$\overline{C}'(x) = -\frac{2000}{x^2} + 0 - 0.0001 = -\frac{2000}{x^2} - 0.0001.$$

10. a. $R(x) = px = x(-0.04x + 800) = -0.04x^2 + 800x$

b. $R'(x) = -0.08x + 800$ c. $R'(5000) = -0.08(5000) + 800 = 400.$

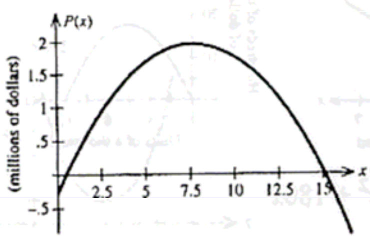
This says that when the level of production is 5000 units the production of the next speaker system will bring an additional revenue of \$400.

11. a. $P(x) = R(x) - C(x) = (-0.04x^2 + 800x) - (200x + 300{,}000) = -0.04x^2 + 600x - 300{,}000.$

b. $P'(x) = -0.08x + 600$

c. $P'(5000) = -0.08(5000) + 600 = 200$ $P'(8000) = -0.08(8000) + 600 = -40.$

d.



The profit realized by the company increases as production increases, peaking at a level of production of 7500 units. Beyond this level of production, the profit begins to fall.

13. a. The revenue function is $R(x) = px = (600 - 0.05x)x = 600x - 0.05x^2$ and the profit function is

$$P(x) = R(x) - C(x)$$
$$= (600x - 0.05x^2) - (0.000002x^3 - 0.03x^2 + 400x + 80{,}000)$$
$$= -0.000002x^3 - 0.02x^2 + 200x - 80{,}000.$$

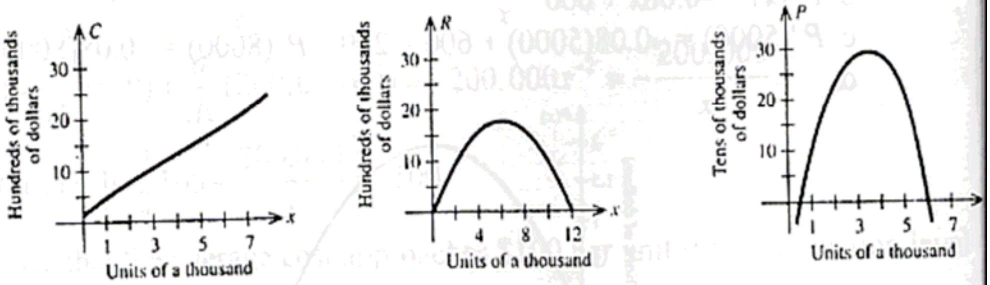
b. $$C'(x) = \frac{d}{dx}(0.000002x^3 - 0.03x^2 + 400x + 80{,}000) = 0.000006x^2 - 0.06x + 400.$$

$$R'(x) = \frac{d}{dx}(600x - 0.05x^2) = 600 - 0.1x.$$

$$P'(x) = \frac{d}{dx}(-0.000002x^3 - 0.02x^2 + 200x - 80{,}000) = -0.000006x^2 - 0.04x + 200.$$

c. $C'(2000) = 0.000006(2000)^2 - 0.06(2000) + 400 = 304$, and this says that at a level of production of 2000 units, the cost for producing the 2001st unit is \$304. $R'(2000) = 600 - 0.1(2000) = 400$ and this says that the revenue realized in selling the 2001st unit is \$400. $P'(2000) = R'(2000) - C'(2000) = 400 - 304 = 96$, and this says that the revenue realized in selling the 2001st unit is \$96.

d.



15. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.000002x^3 - 0.03x^2 + 400x + 80{,}000}{x}$

$= 0.000002x^2 - 0.03x + 400 + \dfrac{80{,}000}{x}.$

a. $\overline{C}'(x) = 0.000004x - 0.03 - \dfrac{80{,}000}{x^2}.$

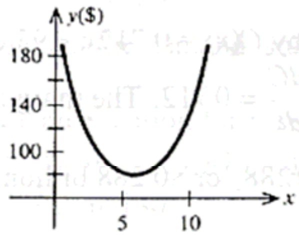
b. $\overline{C}'(5000) = 0.000004(5000) - 0.03 - \dfrac{80{,}000}{5000^2} \approx -0.0132,$

and this says that, at a level of production of 5000 units, the average cost of production is dropping at the rate of approximately a penny per unit.

$\overline{C}'(10{,}000) = 0.000004(10000) - 0.03 - \dfrac{80{,}000}{10{,}000^2} \approx 0.0092,$

and this says that, at a level of production of 10,000 units, the average cost of production is increasing at the rate of approximately a penny per unit.

c.



29. $f(p) = \frac{1}{5}(225 - p^2);\ f'(p) = \frac{1}{5}(-2p) = -\frac{2}{5}p.$

Then the elasticity of demand is given by

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(-\frac{2}{5}p)}{\frac{1}{5}(225 - p^2)} = \frac{2p^2}{225 - p^2}.$$

a. When $p = 8$, $E(8) = \dfrac{2(64)}{225 - 64} = 0.8 < 1$ and the demand is inelastic. When $p = 10$,

$E(10) = \dfrac{2(100)}{225 - 100} = 1.6 > 1$

and the demand is elastic.

b. The demand is unitary when $E = 1$. Solving $\dfrac{2p^2}{225 - p^2} = 1$ we find $2p^2 = 225 - p^2$, $3p^2 = 225$, and $p = 8.66$. So the demand is unitary when $p = 8.66$.

c. Since demand is elastic when $p = 10$, lowering the unit price will cause the revenue to increase.

d. Since the demand is inelastic at $p = 8$, a slight increase in the unit price will cause the revenue to increase.