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5. $f'(x) = \frac{d}{dx}(x^{2.1}) = 2.1x^{1.1}$.

9. $f'(r) = \frac{d}{dr}(\pi r^2) = 2\pi r$.

15. $f'(x) = \frac{d}{dx}(7x^{-12}) = (-12)(7)x^{(-12-1)} = -84x^{-13}$.

17. $f'(x) = \frac{d}{dx}(5x^2 - 3x + 7) = 10x - 3$.

21. $f'(x) = \frac{d}{dx}(0.03x^2 - 0.4x + 10) = 0.06x - 0.4$.

24. $f(x) = x^2 + 2x + 1 - x^{-1}$; $f'(x) = \frac{d}{dx}(x^2 + 2x + 1 - x^{-1}) = 2x + 2 + x^{-2}$.

27. $f'(x) = \frac{d}{dx}(3x^{-1} + 4x^{-2}) = -3x^{-2} - 8x^{-3}$.

29. $f'(t) = \frac{d}{dt}(4t^{-4} - 3t^{-3} + 2t^{-1}) = -16t^{-5} + 9t^{-4} - 2t^{-2}$.

35. a. $f'(x) = \frac{d}{dx}(2x^3 - 4x) = 6x^2 - 4$. $f'(-2) = 6(-2)^2 - 4 = 20$.
b. $f'(0) = 6(0) - 4 = -4$. c. $f'(2) = 6(2)^2 - 4 = 20$.

37. The given limit is $f'(1)$ where $f(x) = x^3$. Since $f'(x) = 3x^2$, we have

$$\lim_{h\to 0} \frac{(1+h)^3 - 1}{h} = f'(1) = 3$$

39. Let $f(x) = 3x^2 - x$. Then

$$\lim_{h\to 0} \frac{3(2+h)^2 - (2+h) - 10}{h} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

because $f(2 + h) - f(2) = 3(2 + h)^2 - (2 + h) - [3(4) - 2]$
$= 3(2 + h)^2 - (2 + h) - 10$.
But the last limit is $f'(2)$. Since $f'(x) = 6x - 1$, we have $f'(2) = 11$.

Therefore, $\lim_{h\to 0} \frac{3(2+h)^2 - (2+h) - 10}{h} = 11$.

42. $f(x) = -\frac{5}{3}x^2 + 2x + 2$. $f'(x) = -\frac{10}{3}x + 2$. The slope is $f'(-1) = \frac{10}{3} + 2 = \frac{16}{3}$.

An equation of the tangent line is

$$y + \frac{5}{3} = \frac{16}{3}(x+1) \quad \text{or} \quad y = \frac{16}{3}x + \frac{11}{3}.$$

47. a. $f(x) = x^3 + 1$. The slope of the tangent line at any point $(x, f(x))$ on the graph of f is $f'(x) = 3x^2$. At the point(s) where the slope is 12, we have $3x^2 = 12$, or $x = \pm 2$. The required points are (-2,-7) and (2,9).

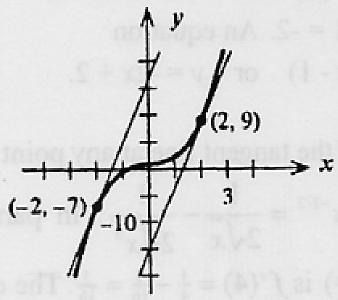
b. The tangent line at (-2,-7) has equation

$$y - (-7) = 12[x - (-2)], \quad \text{or} \quad y = 12x + 17,$$

and the tangent line at (2,9) has equation

$$y - 9 = 12(x - 2), \quad \text{or} \quad y = 12x - 15.$$

c.



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2. $f(x) = 3x^2(x - 1)$

$$f'(x) = 3x^2 \frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(3x^2) = 3x^2 + (x-1)(6x) = 9x^2 - 6x.$$

4. $f(x) = (2x + 3)(3x - 4)$

$$f'(x) = (2x+3)\frac{d}{dx}(3x-4) + (3x-4)\frac{d}{dx}(2x+3)$$

$$= (2x+3)(3) + (3x-4)(2) = 12x + 1.$$

7. $f(x) = (x^3 - 1)(x + 1)$.

$$f'(x) = (x^3 - 1)\frac{d}{dx}(x+1) + (x+1)\frac{d}{dx}(x^3 - 1)$$

$$= (x^3 - 1)(1) + (x+1)(3x^2) = 4x^3 + 3x^2 - 1.$$

11. $f(x) = (5x^2 + 1)(2\sqrt{x} - 1)$

$$f'(x) = (5x^2 + 1)\frac{d}{dx}(2x^{1/2} - 1) + (2x^{1/2} - 1)\frac{d}{dx}(5x^2 + 1)$$

$$= (5x^2 + 1)(x^{-1/2}) + (2x^{1/2} - 1)(10x)$$

$$= 5x^{3/2} + x^{-1/2} + 20x^{3/2} - 10x = \frac{25x^2 - 10x\sqrt{x} + 1}{\sqrt{x}}.$$

15. $f(x) = \frac{1}{x-2}$. $f'(x) = \frac{(x-2)\frac{d}{dx}(1) - (1)\frac{d}{dx}(x-2)}{(x-2)^2} = \frac{0-1(1)}{(x-2)^2} = -\frac{1}{(x-2)^2}$.

19. $f(x) = \frac{1}{x^2+1}$.

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}(1) - (1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(0) - 1(2x)}{(x^2+1)^2} = -\frac{2x}{(x^2+1)^2}.$$

22. $f(x) = \frac{x^3-2}{x^2+1}$.

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}(x^3-2) - (x^3-2)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(3x^2) - (x^3-2)(2x)}{(x^2+1)^2} = \frac{x(x^3+3x+4)}{(x^2+1)^2}.$$

24. $f(x) = \frac{x^2+1}{\sqrt{x}}$.

$$f'(x) = \frac{x^{1/2}(2x) - (x^2+1)(\frac{1}{2}x^{-1/2})}{x} = \frac{\frac{1}{2}x^{-1/2}[4x^2 - (x^2+1)]}{x} = \frac{3x^2-1}{2x^{3/2}}.$$

27. $f(x) = \frac{(x+1)(x^2+1)}{x-2} = \frac{(x^3+x^2+x+1)}{x-2}$.

$$f'(x) = \frac{(x-2)\frac{d}{dx}(x^3+x^2+x+1) - (x^3+x^2+x+1)\frac{d}{dx}(x-2)}{(x-2)^2}$$

$$= \frac{(x-2)(3x^2+2x+1) - (x^3+x^2+x+1)}{(x-2)^2}$$

$$= \frac{3x^3+2x^2+x-6x^2-4x-2-x^3-x^2-x-1}{(x-2)^2} = \frac{2x^3-5x^2-4x-3}{(x-2)^2}.$$

30. $f(x) = \frac{x+\sqrt{3x}}{3x-1}$.

$$f'(x) = \frac{(3x-1)(1+\frac{1}{2}\sqrt{3}x^{-1/2}) - (x+\sqrt{3}x^{1/2})(3)}{(3x-1)^2}$$

$$= \frac{3x+\frac{3}{2}\sqrt{3}x^{1/2} - 1 - \frac{1}{2}\sqrt{3}x^{-1/2} - 3x - 3\sqrt{3}x^{1/2}}{(3x-1)^2} = -\frac{3\sqrt{3}x + 2\sqrt{x} + \sqrt{3}}{2\sqrt{x}(3x-1)^2}.$$

35. $f(x) = (2x-1)(x^2+3)$

$$f'(x) = (2x-1)\frac{d}{dx}(x^2+3) + (x^2+3)\frac{d}{dx}(2x-1)$$
$$= (2x-1)(2x) + (x^2+3)(2) = 6x^2 - 2x + 6 = 2(3x^2 - x + 3).$$

At $x = 1, f'(1) = 2[3(1)^2 - (1) + 3] = 2(5) = 10.$

36. $f(x) = \dfrac{2x+1}{2x-1}.$

$$f'(x) = \frac{(2x-1)\frac{d}{dx}(2x+1) - (2x+1)\frac{d}{dx}(2x-1)}{(2x-1)^2}$$
$$= \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2} = \frac{4x-2-4x-2}{(2x-1)^2} = -\frac{4}{(2x-1)^2}.$$

At $x = 2,\ f'(2) = \dfrac{-4}{[2(2)-1]^2} = -\dfrac{4}{9}.$

39. $f(x) = (x^3+1)(x^2-2).$

$$f'(x) = (x^3+1)\frac{d}{dx}(x^2-2) + (x^2-2)\frac{d}{dx}(x^3+1)$$
$$= (x^3+1)(2x) + (x^2-2)(3x^2).$$

The slope of the tangent line at (2,18) is $f'(2) = (8+1)(4) + (4-2)(12) = 60.$
An equation of the tangent line is $y - 18 = 60(x - 2)$, or $y = 60x - 102.$

47. $f(x) = (x^2+6)(x-5)$

$$f'(x) = (x^2+6)\frac{d}{dx}(x-5) + (x-5)\frac{d}{dx}(x^2+6)$$
$$= (x^2+6)(1) + (x-5)(2x) = x^2 + 6 + 2x^2 - 10x = 3x^2 - 10x + 6.$$

At a point where the slope of the tangent line is -2, we have
$f'(x) = 3x^2 - 10x + 6 = -2.$
This gives $3x^2 - 10x + 8 = (3x-4)(x-2) = 0$. So $x = \frac{4}{3}$ or $x = 2$.
Since $f(\frac{4}{3}) = (\frac{16}{9} + 6)(\frac{4}{3} - 5) = -\frac{770}{27}$ and $f(2) = (4+6)(2-5) = -30$,
the required points are $(\frac{4}{3}, -\frac{770}{27})$ and $(2, -30)$.