21. $f(x) = -1/x$. We first compute $f'(x)$ using the four-step process.

Step 1 $f(x+h) = -\dfrac{1}{x+h}$

Step 2 $f(x+h) - f(x) = -\dfrac{1}{x+h} + \dfrac{1}{x} = \dfrac{-x + (x+h)}{x(x+h)} = \dfrac{h}{x(x+h)}$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\dfrac{h}{x(x+h)}}{h} = \dfrac{1}{x(x+h)}$

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{1}{x(x+h)} = \dfrac{1}{x^2}$.

The slope of the tangent line is $f'(3) = 1/9$. Therefore, a required equation is

$$y - (-\tfrac{1}{3}) = \tfrac{1}{9}(x-3) \quad \text{or} \quad y = \tfrac{1}{9}x - \tfrac{2}{3}.$$

23. a. $f(x) = 2x^2 + 1$. We use the four-step process.

Step 1 $f(x+h) = 2(x+h)^2 + 1 = 2x^2 + 4xh + 2h^2 + 1$

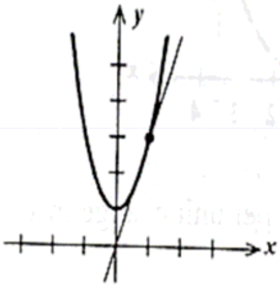
Step 2 $f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 + 1) - (2x^2 + 1) = 4xh + 2h^2$
$= h(4x + 2h)$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x+2h)}{h} = 4x + 2h$

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (4x + 2h) = 4x$

b. The slope of the tangent line is $f'(1) = 4(1) = 4$. Therefore, an equation is $y - 3 = 4(x - 1)$ or $y = 4x - 1$.

c.



32. a. Its height after 40 seconds is

$$f(40) = \tfrac{1}{2}(40)^2 + \tfrac{1}{2}(40) = 820 \qquad [\text{Writing } f(t) = \tfrac{1}{2}t^2 + \tfrac{1}{2}t.]$$

b. Its average velocity over the interval [0,40] is

$$\frac{f(40) - f(0)}{40 - 0} = \frac{820 - 0}{40} = 20.5, \text{ or } 20.5 \text{ ft/sec.}$$

c. Its velocity at time t is

$$v(t) = \lim_{h\to 0}\frac{f(t+h) - f(t)}{h} = \lim_{h\to 0}\frac{\frac{1}{2}(t+h)^2 + \frac{1}{2}(t+h) - (\frac{1}{2}t^2 + \frac{1}{2}t)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{2}t^2 + th + \frac{1}{2}h^2 + \frac{1}{2}t + \frac{1}{2}h - \frac{1}{2}t^2 - \frac{1}{2}t}{h}$$

$$= \lim_{h\to 0}\frac{th + \frac{1}{2}h^2 + \frac{1}{2}h}{h} = \lim_{h\to 0}(t + \tfrac{1}{2}h + \tfrac{1}{2}) = t + \tfrac{1}{2}.$$

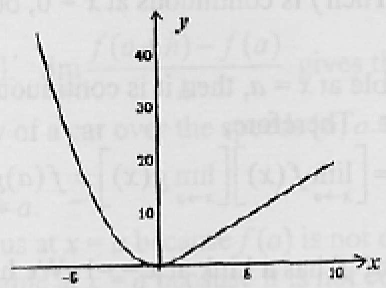
In particular, the velocity at the end of 40 seconds is

$$v(40) = 40 + \tfrac{1}{2}, \text{ or } 40\tfrac{1}{2} \text{ ft/sec.}$$

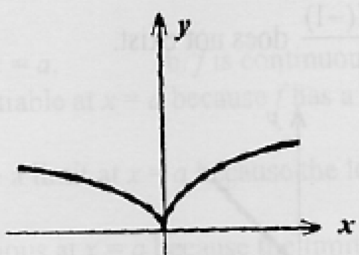
57. For continuity, we require that $f(1) = 1 = \lim_{x\to 1^+}(ax + b) = a + b$, or $a + b = 1$.

In order that the derivative exist at $x = 1$, we require that $\lim_{x\to 1^-} 2x = \lim_{x\to 1^+} a$, or $2 = a$.

Therefore, $b = -1$ and so $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$. The graph of f follows.



58. f is continuous at $x = 0$, but $f'(0)$ does not exist because the graph of f has a vertical tangent line at $x = 0$. The graph of f follows.



59. We have $f(x) = x$ if $x > 0$ and $f(x) = -x$ if $x < 0$. Therefore, when $x > 0$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{x+h-x}{h} = \lim_{h\to 0}\frac{h}{h} = 1,$$

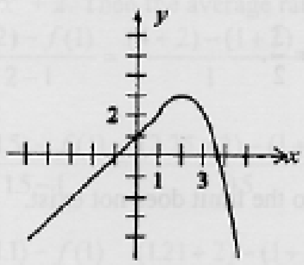
and when $x < 0$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-x-h-(-x)}{h} = \lim_{h\to 0}\frac{-h}{h} = -1.$$

Since the right–hand limit does not equal the left–hand limit, we conclude that $\lim_{h\to 0} f(x)$ does not exist.

p158

4.



15. $\lim_{x\to 1^-}\frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1^-}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-}\frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-}\frac{1}{\sqrt{x}+1} = \frac{1}{2}.$

16. $\lim_{x\to\infty}\frac{x^2}{x^2-1} = \lim_{x\to\infty}\frac{1}{1-\frac{1}{x^2}} = 1.$

17. $\lim_{x\to-\infty}\frac{x+1}{x} = \lim_{x\to-\infty}\left(1+\frac{1}{x}\right) = 1.$

19. $\lim_{x\to-\infty}\frac{x^2}{x+1} = \lim_{x\to-\infty} x\cdot\frac{1}{1+\frac{1}{x}} = -\infty$, so the limit does not exist.

21. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+2) = 4;$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(4-x) = 2.$

Therefore, $\lim_{x\to 2} f(x)$ does not exist.