Problems of the matrix

1. Let $A=\begin{pmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 5 & -2\\ 2 & 2 & -1\end{pmatrix}$. Find $A+B$, $3B$, $2A+B$, $A-2B$, A^T, $(A+B)^T$.
2. Write down the row vectors and column vectors of matrices A, B in problem 1.
3. Let $A=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $B=\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$. Find AB and BA.
4. Let $A=\begin{pmatrix}2 & -1 & 1\\ 3 & 0 & -1\end{pmatrix}$, $B=\begin{pmatrix}1 & 1 & 0\\ 2 & 1 & -1\\ 3 & 1 & 5\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2\\ 3 & 1\\ -1 & 4\end{pmatrix}$. Find $(AB)C$ and $A(BC)$.
5. Let A be an $n\times n$ matrix. Define the trace of A to be the sum of the diagonal elements. Thus, if $A=(a_{ij})$, then

$$\mathrm{tr}(A)=\sum_{i=1}^{n} a_{ii}$$

   Compute the trace of the matrix $A=\begin{pmatrix}1 & 7 & 3\\ -1 & 5 & 2\\ 2 & 3 & -4\end{pmatrix}$.
6. Find the rank of the following matrices:

   (1) $\begin{pmatrix}2 & 1 & 3\\ 7 & 2 & 0\end{pmatrix}$ (2) $\begin{pmatrix}1 & 2 & -3\\ -1 & -2 & 3\\ 4 & 8 & -12\\ 0 & 0 & 0\end{pmatrix}$ (3) $\begin{pmatrix}-1 & 0 & 1\\ 0 & 2 & 3\\ 0 & 0 & 7\end{pmatrix}$
7. Computer the following determinants:

   (1) $\begin{vmatrix}2 & 1 & 2\\ 0 & 3 & -1\\ 4 & 1 & 1\end{vmatrix}$ (2) $\begin{vmatrix}2 & 4 & 3\\ -1 & 3 & 0\\ 0 & 2 & 1\end{vmatrix}$ (3) $\begin{vmatrix}1 & 1 & -2 & 4\\ 0 & 1 & 1 & 3\\ 2 & -1 & 1 & 0\\ 3 & 1 & 2 & 5\end{vmatrix}$ (4) $\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$
8. Solve the following systems of linear equations:

   (1) $\begin{cases}2x-y+z=1\\ x+3y-2z=0\\ 4x-3y+z=2\end{cases}$ (2) $\begin{cases}4x+y+z+w=1\\ x-y+2z-3w=0\\ 2x+y+3z+5w=0\\ x+y-z-w=2\end{cases}$

9. Find the inverse of the following matrices:

(1) $\begin{pmatrix} 3 & -1 \\ 1 & 4 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & -1 & 3 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 0 & 2 & -1 \\ 4 & 2 & 1 & 16 \end{pmatrix}$

10. Find the relation between b_1, b_2, and b_3 such that the system $\begin{cases} x_1 + 5x_2 + 2x_3 = b_1 \\ 2x_1 + x_2 + x_3 = b_2 \\ x_1 + 2x_2 + x_3 = b_3 \end{cases}$ has a solution.