Practical Examples of Operations Research

本章列出一些作業研究所探討的一些實例，希望透過這一些實例的介紹，可以讓讀者有一些簡單的概念。

Example 1 (Giapetto's Woodcarving)

Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains. A soldier sells for $27 and uses $10 worth of raw materials(原物料). Each soldier that is manufactured increases Giapetto's variable labor and overhead(經常的)costs by $14. A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto's variable labor and overhead costs by $10.The manufacture of wooden soldier and train requires two types of skilled labor: carpentry(木工)and finishing(最後的修整). A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing labor and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most(至多) 40 soldiers are bought each week. Giapetto wants to maximize weekly profit.

Solution:

We define

$x_1 =$ number of soldiers produced each week

$x_2 =$ number of trains produced each week

Then, the optimization model is

$$\max \quad z = (27-10-14)x_1 + (21-9-10)x_2 = 3x_1 + 2x_2$$

subject to (s.t.)

$$\begin{aligned} 2x_1 + \ x_2 &\le 100 \\ x_1 + \ x_2 &\le 80 \\ x_1 \qquad &\le 40 \\ x_1, \ x_2 &\ge 0 \end{aligned}$$

Hence, we find that $(x_1 = 20, x_2 = 60)$ is the optimal solution, and the optimal value of z is $z = 3(20) + 2(60) = 180$.

Example 2 (Dorian Auto)

Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach(爭取)these groups, Dorian Auto has embarked on(從事) an ambitious TV advertising campaign(廣告活動) and has decided to purchase 1-munute commercial spots(節目) on two types of programs: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy and costs $50000, and a 1-minute football and costs $100000. Dorian would like the commercial to be seen by at least 28 million high-income women and 24 million high-income men. Determine how Dorian Auto can meet its advertising requirement at minimum cost.

Solution:

We define

$x_1 =$ number of 1-minute comedy advertising purchased

$x_2 =$ number of 1-minute football advertising purchased

Then, the optimization model is

$$\min \quad z = 50x_1 + 100x_2$$

subject to (s.t.)

$$7x_1 + 2x_2 \geq 28$$
$$2x_1 + 12x_2 \geq 24$$
$$x_1, x_2 \geq 0$$

Hence, we find that $(x_1 = 3.6, x_2 = 1.4)$ is the optimal solution, and the optimal value of z is $z = 50(3.6) + 100(1.4) = 320 = \$320000$.

Example 3: (Manufacturing)

An auto company manufactures cars and trucks. Each vehicle(車輛) must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, then 40 per day could be painted. If the paint shop were only painting cars, then 60 per day could be painted. If the body shop were only producing cars, then it could process 50 per day. If the body shop were only producing trucks, then it could process 50 per day. Each truck contributes \$300 to profit, and each car contributes \$200 to profit. Determine a daily production schedule that will maximize the company's profits.

Solution:

We define

$x_1 =$ number of trucks produced daily

$x_2 =$ number of cars produced daily

Then, the optimization model is

$$\min \quad z = 3x_1 + 2x_2$$

subject to (s.t.)

$$\frac{1}{40}x_1 + \frac{1}{60}x_2 \leq 1$$
$$\frac{1}{50}x_1 + \frac{1}{50}x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

Hence, we find that $(x_1 = 40, x_2 = 0)$ is the optimal solution, and the optimal value of z is $z = 3(40) + 2(0) = 120$.

Example 4 (Diet Problem)

My diet requires that all the food I eat come from one of the four "basic food groups" (chocolate cake, ice cream, soda, and cheesecake). At present, the following four foods are available for consumptions(消耗量): brownies(巧克力小方餅), chocolate ice cream, cola, and pineapple(鳳

梨)cheesecake. Each brownies costs \$50, each scoop of chocolate ice cream costs \$20, each bottle of cola costs \$30, and each piece of pineapple cheesecake costs \$80. Each day, I must ingest(攝取)at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional(營養的)contest per unit of each food is shown in below Table. Determine how to satisfy my daily nutritional requirement at minimum cost.

Table Nutritional Values for Diet

Type of Food	Calories	Chocolate (Ounces)	Sugar (Ounces)	Fat (Ounces)
Brownies	400	3	2	2
Chocolate ice cream (1 scoop)	200	2	2	4
Cola (1 bottle)	150	0	4	1
Pineapple cheesecake (1 piece)	500	0	4	5

Solution:

We define

$x_1 =$ number of brownies eaten daily

$x_2 =$ number of scoops of chocolate ice cream eaten daily

$x_3 =$ bottles of cola drunk daily

$x_4 =$ pieces of pineapple cheesecake eaten daily

Then, the optimization model is

$$\begin{aligned} \min \quad & z = 50x_1 + 20x_2 + 30x_3 + 80x_4 \\ \text{subject to (s.t.)} \quad & \\ & 400x_1 + 200x_2 + 150x_3 + 500x_4 \ge 500 \\ & 3x_1 + 2x_2 \ge 6 \\ & 2x_1 + 2x_2 + 4x_3 + 4x_4 \ge 10 \\ & 2x_1 + 4x_2 + x_3 + 5x_4 \ge 8 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

Hence, we find that $(x_1 = 0, x_2 = 3, x_3 = 1, x_4 = 0)$ is the optimal solution, and the optimal value of z is $z = 90$.

Example 5 (Post Office Problem-Work-Scheduling Problem)

A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is given in below Table. Union(工會) rules state that each full-time employees must work five consecutive(連續) days and then receive two days off. The post office wants to meet(滿足)its daily requirements using only full-time employees. Determine that the post office can use to minimize the number of full-time employees who must be hired.

Solution:

We define

$x_i =$ number of employees beginning work on day i.

Then, the optimization model is

$$\min \quad z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

subject to (s.t.)

$$\begin{aligned}
x_1 \qquad\qquad + x_4 + x_5 + x_6 + x_7 &\ge 17\\
x_1 + x_2 \qquad\qquad + x_5 + x_6 + x_7 &\ge 13\\
x_1 + x_2 + x_3 + \qquad\qquad x_6 + x_7 &\ge 15\\
x_1 + x_2 + x_3 + x_4 \qquad\qquad + x_7 &\ge 19\\
x_1 + x_2 + x_3 + x_4 + x_5 \qquad\qquad &\ge 14\\
x_2 + x_3 + x_4 + x_5 + x_6 \qquad &\ge 16\\
x_3 + x_4 + x_5 + x_6 + x_7 &\ge 17\\
x_1, x_2, x_3, x_4, x_5, x_6, x_7 &\ge 0
\end{aligned}$$

Hence, we find that $(x_1 = 4, x_2 = 4, x_3 = 2, x_4 = 6, x_5 = 0, x_6 = 4, x_7 = 3)$ is the optimal solution, and the optimal value of z is $z = 23$.

Table Requirements for Post Office

Day	Number of Full-time Employees Required
1 = Monday	17
2 = Tuesday	13
3 = Wednesday	15
4 = Thursday	19
5 = Friday	14
6 = Saturday	16
7 = Sunday	11

Example 6 (Project Selection)

Star Oil Company is considering five different investment opportunities. The cash outflows and net present values (in millions of dollars) are given in below Table. Star Oil has \$40 million available for investment now (time 0); it estimates that one year from now (time 1) \$20 million will be available for investment. Star Oil may purchases any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly. For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 0, and a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 1. The one-fifth share of investment 3 would yield an NPV of $\frac{1}{5}(16) = \$3.2$ million. Star Oil wants to maximize the NPV that can be obtained by investing in investments 1-5. Assume that any funds left over at time 0 cannot be used at time.

Solution:

We define

$x_i =$ fraction of investment i purchased any Star Oil ($i = 1,2,3,4,5$)

Then, the optimization model is

$$\max \quad z = 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$$

subject to (s.t.)

$$\begin{aligned} 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 &\le 40 \\ 3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 &\le 20 \\ x_1 &\le 1 \\ x_2 &\le 1 \\ x_3 &\le 1 \\ x_4 &\le 1 \\ x_5 &\le 1 \\ x_1, x_2, x_3, x_4, x_5 &\ge 0 \end{aligned}$$

Hence, we find that $(x_1 = 1, x_2 = 0.201, x_3 = 1, x_4 = 1, x_5 = 0.288)$ is the optimal solution, and the optimal value of z is $z = 57.449$.

Table Cash Flows and Net Present Value for investment in Capital Budgeting

	Investment ($)				
	1	2	3	4	5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	34
NPV	13	16	16	14	39

Example 7 (Short-Term Financial Planning)

Semicond is a small electronic company that manufactures type recorders and radios. The per-unit labor costs, raw material costs, and selling price of each product are given in below Table. On December 1, 2002, Semicond has available raw material that is sufficient to manufacture 100 type recorders and 100 radios. On the same date, the company's balance sheet(資產負債表) is as shown in below Table, and Semicond's asset-liability (called the current ratio) is $\frac{20000}{10000} = 2$.

Semicond must determine how many type recorders and radios should be produced during December. Demand is large enough to ensure that all goods produced will be sold. All sales are on credit(憑信用卡), however, and payment for goods produced in December will not be received until February 1, 2003. During December, Semicond will collect $2000 in accounts receivable, and Semicond must pay off $1000 of the outstanding(未償貸款) loan and a monthly rent $1000. On January 1, 2003, Semicond will receive a shipment of raw material worth $2000, which will be paid for a February 1, 2003. Semicond's management has decided that the cash balance on January 1, 2003, must be at least $4000. Also, Semicond's bank requires that the current ratio at the beginning of January be at least 2. To maximize the contribution to profit from December production, what should Semicond during December?

Solution:

We define

$x_1 =$ number of type recorders produced during December

$x_2 =$ number of radios produced during December

Then, the optimization model is

$$\max \quad z = 20x_1 + 15x_2$$

subject to (s.t.)

$$\begin{aligned} x_1 \quad\quad &\le 100 \\ x_2 &\le 100 \\ 50x_1 + 35x_2 &\le 6000 \\ 20x_1 + 15x_2 &\ge 2000 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Hence, we find that $(x_1 = 50, x_2 = 100)$ is the optimal solution, and the optimal value of z is $z = 2500$.

Table Cost Information for Semicond

	Type Recorder	Radio
Selling price	$100	$90
Labor cost	$50	$35
Raw material cost	$30	$40

Table Balance Sheet for Semicond

	Assets	Liabilities
Cash	$10000	
Accounts receivable	$3000	
Inventory outstanding	$7000	
Bank loan		$10000

Example 8 (Brute Production Process)

Rylon Corporation manufacturers Brute and Chanelle perfumes(香水). The raw material needed to manufacture each type of perfume can be purchased $3 per pound. Processing 1 lb of raw material requires 1 hour of laboratory time. Each pound of processed raw material yields 3 oz of Regular Brute Perfume and 4 oz of Regular Chanelle Perfume. Regular Brute can be sold for $7/oz and Regular Chanelle for $6/oz. Rylon also has the option of further processing Regular Brute and Regular Chanelle to produce Luxury Brute, sold at $18/oz, and Luxury Chanelle, sold at $14/oz. Each ounce of Regular Brute processed further requires an additional(額外) 3 hours of laboratory time and $4 processing cost and yields 1 oz of Luxury Brute. Each ounce of Regular Chanelle processed further requires an additional 2 hours of laboratory time and $4 processing cost and yields 1 oz of Luxury Brute. Each year, Rylon has 6000 hours of laboratory time available and can purchase up 4000 lb of raw material. Formulate an LP that can be used to determine how Rylon can maximize profit. Assume that the cost of the laboratory hours is a fixed cost.

Solution:

We define

$x_1 =$ number of ounces of Regular Brute sold annual

$x_2 =$ number of ounces of Luxury Brute sold annual

$x_3 =$ number of ounces of Regular Chanelle sold annual

$x_4 =$ number of ounces of Luxury Chanelle sold annual

$x_5 =$ number of pounds of raw material purchased annual

Then, the optimization model is

$$\max \quad z = 7x_1 + 14x_2 + 6x_3 + 10x_4 - 3x_5$$

subject to (s.t.)

$$\begin{aligned} x_5 &\le 4000 \\ 2x_2 + 2x_4 + x_5 &\le 6000 \\ x_1 + x_2 - 3x_5 &= 0 \\ x_3 + x_4 - 4x_5 &= 0 \\ x_i \ge 0 (i &= 1,2,3,4,5) \end{aligned}$$

The optimal solution is $x_1 = 11333.333$, $x_2 = 666.667$, $x_3 = 16000$, $x_4 = 0$, $x_5 = 4000$, and $z = 172666.667$.

Example 9 (Sailco Inventory)

Sailco Corporation must determine how many sailboats should be produced during each of the next four quarters. The demand during each of the next four quarters is as follows: first quarter, 40 sailboats; second quarter, 60 sailboats; third quarter, 75 sailboats; fourth quarter, 25 sailboats. Sailco must meet demands on time. At the beginning of the first quarter, Sailco has an inventory of 10 sailboats. At the beginning of each quarter, Sailco must decide how many sailboats should be produced during that quarter. For simplicity, we assume that sailboats manufactured during a quarter can be used to meet demand for that quarter. During each quarter, Sailco can produce up to 40 sailboats with regular-time labor at a total cost of $400 per sailboat. By having employees work overtime during a quarter, Sailco can produce additional sailboats with overtime labor at a total cost of $450 per sailboat.

At the end of each quarter (after production has occurred and the current quarter’s demand has been satisfied), a carrying or holding cost of $20 per sailboat is incurred. Use linear programming to determine a production schedule to minimize the sum of production and inventory costs during the next four quarters.

Solution:

We define

$x_t =$ number of sailboats produced by regular-time labor during quarter t.

$y_t =$ number of sailboats produced by overtime labor during quarter t.

$i_t =$ number of sailboats on hand at end of quarter t.

Then, the optimization model is

$$\min \quad z = 400\sum_{t=1}^{4} x_t + 450\sum_{t=1}^{4} y_t + 20\sum_{t=1}^{4} i_t$$

subject to (s.t.)

$$\begin{aligned} i_1 &= 10 + x_1 + y_1 - 40 \\ i_2 &= i_1 + x_2 + y_2 - 60 \\ i_3 &= i_2 + x_3 + y_3 - 75 \\ i_4 &= i_3 + x_4 + y_4 - 25 \\ 0 \le x_t &\le 40, y_t \ge 0, i_t \ge 0 \end{aligned}$$

The optimal solution is $x_1 = x_2 = x_3 = 40$; $x_4 = 25$; $y_1 = 0$; $y_2 = 10$; $y_3 = 35$; $y_4 = 0$;

$i_1 = 10$; $i_2 = i_3 = i_4 = 0$, and $z = 78450$.

Example 10 (Transportation Problem)

One of the main products of the P&T COMPANY is canned peas. The peas are prepared at three canneries and then shipped by truck to four distributing warehouses in the western United States. Because the shipping costs are a major expense, management is initiating a study to reduce them as much as possible. For the upcoming season, an estimate has been made of the output from each cannery, and each warehouse has been allocated a certain amount from the total supply of peas. This information (in units of truckloads), along with the shipping cost per truckload for each cannery-warehouse combination, is given in Table.

		Shipping Cost ($) per Truckload				
		Warehouse				
		1	2	3	4	Output
	1	464	513	654	867	75
Cannery	2	352	416	690	791	125
	3	995	682	388	685	100
Allocation		80	65	70	85	

The problem is to determine which plan for assigning these shipments to the various cannery-warehouse combinations would minimize the total shipping cost.

Solution:

We define

$x_{ij} =$ number of truckloads to be shipped from cannery i to warehouse j.

Then, the optimization model is

$$\min \quad \begin{aligned} z = {} & 464x_{11} + 513x_{12} + 654x_{13} + 867x_{14} \\ & + 352x_{21} + 416x_{22} + 690x_{23} + 791x_{24} \\ & + 995x_{31} + 682x_{32} + 388x_{33} + 685x_{34} \end{aligned}$$

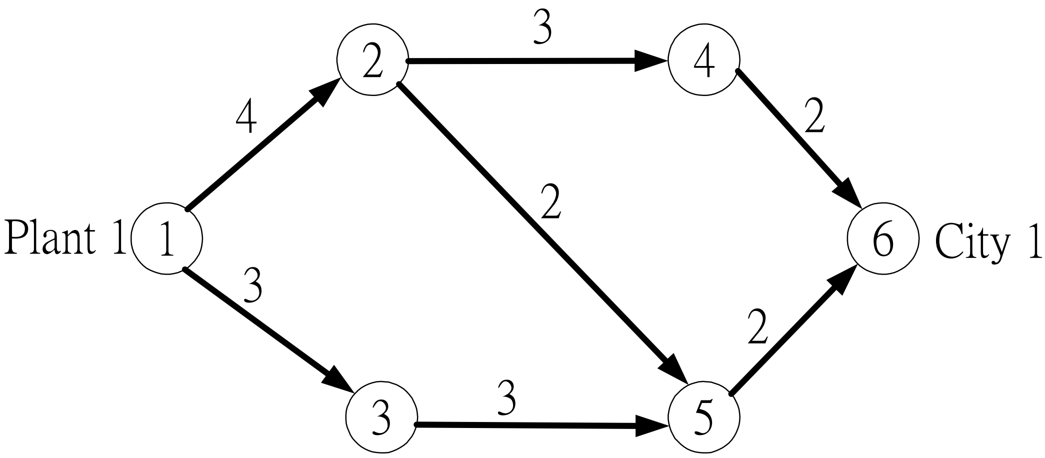
subject to (s.t.)

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} = 75 \\ & x_{21} + x_{22} + x_{23} + x_{24} = 125 \\ & x_{31} + x_{32} + x_{33} + x_{34} = 100 \\ & x_{11} + x_{21} + x_{31} = 80 \\ & x_{12} + x_{22} + x_{32} = 65 \\ & x_{13} + x_{23} + x_{33} = 70 \\ & x_{14} + x_{24} + x_{34} = 85 \\ & x_{ij} \ge 0, (i = 1,2,3;\ j = 1,2,3,4) \end{aligned}$$

The optimal solution is $x_{12} = 20; x_{14} = 55; x_{21} = 80; x_{22} = 45; x_{33} = 70; x_{34} = 30; z = 152535$.

Example 11 (Shortest Path)

Suppose that when power is sent from plant 1 (node 1) to city 1 (node 6), it must pass through relay substations (nodes 2-5). For any pair of nodes between which power can be transported, Figure 2 gives the distance between the nodes. Thus, substation 2 and 4 are 3 miles apart, and power cannot be sent between substation 4 and 5. Powerco wants the power sent from plant 1to city 1 to travel the minimum possible distance, so it must find the shortest path in Figure 2 that joins node 1 to node 6.



Example 12 (Game)

在我國古代，”齊王賽馬”就是一個典型的對策論的例子。

戰國時期，齊王有一天提出要與田忌進行賽馬。雙方約定：從各自的上、中、下三各等級的馬各選一匹參賽；每匹馬只能參賽一次；每一次比賽雙方各出一匹馬，輸者要付給贏者千金，已經知道，在同等級的馬中，田忌的馬不如齊王的馬，而如果田忌的馬比齊王的馬高一等級，則田忌的馬可以取勝。

齊王的策略 \ 齊王的報酬 \ 田忌的策略	$[\beta_1]$(上中下)	$[\beta_2]$(上下中)	$[\beta_3]$(中上下)	$[\beta_4]$(中下上)	$[\beta_5]$(下中上)	$[\beta_6]$(下上中)
$[\alpha_1]$(上中下)	3	1	1	1	1	-1
$[\alpha_2]$(上下中)	1	3	1	1	-1	1
$[\alpha_3]$(中上下)	1	-1	3	1	1	1
$[\alpha_4]$(中下上)	-1	1	1	3	1	1
$[\alpha_5]$(下中上)	1	1	-1	1	3	1
$[\alpha_6]$(下上中)	1	1	1	1	1	3

Which strategy should each player select?

Example 13 (Newsboy Problem)

Sales of ski equipment and apparel depend heavily on snow conditions. The Ski Bum, a small discount retailer of ski equipment near some of the large ski areas of New Hampshire and Vermont, faces the difficult task of ordering a particular line of ski gloves in the month of May, well before the ski season begins. The selling price for these gloves is $50.30 per pair, while the cost to the retailer is only $35.10. The retailer can usually buy additional gloves from competitors at their retail price of $60. Gloves left over at the end of the short selling season are sold at a special discount price of $25 per pair. The owner and manager of the store assumes that demand is approximately normally distributed with a mean of 900 gloves and a standard derivation of 122 gloves. What is the ordering number of the retailer?

Example 14 (How Many Repairers?)

SIMULATION, INC., a small company that makes gidgets for analog computers, has 10 gidget-making machines. However, because these machines break down and require repair frequently, the company has only enough operators to operate eight machines at a time, so two machines are available on a standby basis for use while other machines are down. Thus, eight machines are always operating whenever no more than two machines are waiting to be repaired, but the number of operating machines is reduced by 1 for each additional machine waiting to be repaired.

The time until any given operating machine breaks down has an exponential distribution, with a mean of 20 days. (A machine that is idle on a standby basis cannot break down.) The time required to repair a machine also has an exponential distribution, with a mean of 2 days. Until now the company has had just one repairer to repair these machines, which has frequently resulted in reduced productivity because fewer than eight machines are operating. Therefore, the company is considering hiring a second repairer, so that two machines can be repaired simultaneously.

Thus, the queueing system to be studied has the repairers as its servers and the machines requiring repair as its customers, where the problem is to choose between having one or two servers.

Example 15 (Data Envelopment Analysis; DEA)

Often we wonder if a university, hospital, restaurant, or other business is operating efficient. The Data Envelopment Analysis (DEA) method can be used to answer this question. Our presentation is based on Callen (1991). To illustrate how DEA works, let’s consider a group of three hospitals. To simplify matters, we assume that each hospital “converts” two inputs into three different outputs. The two inputs used by each hospital are

Input 1 = capital (measured by the number of hospital beds)

Input 2 = labor (measured in thousands of labor hours used during a month)

The outputs produced by each hospital are

Output 1 = hundreds of patient-day during month for patients under age 14

Output 2 = hundreds of patient-day during month for patients between 14 and 65

Output 3 = hundreds of patient-day during month for patients over 65

Suppose that the inputs and outputs for the three hospitals are given in Table.

	Inputs		Outputs		
Hospital	1	2	1	2	3
1	5	14	9	4	16
2	8	15	5	7	10
3	7	12	4	9	13

To determine whether a hospital is efficient, let's define

$t_r =$ price value of one unit of output r

$w_s =$ cost of one unit of input s

The **efficiency** of hospital i is defined to be

$$\frac{\text{value of hospital } i\text{'s outputs}}{\text{cost of hospital } i\text{'s inouts}}$$

Hospital 1 LP

$$\begin{aligned}
\max\ & z = 9t_1 + 4t_2 + 16t_3 \\
\text{s.t.}\ & -9t_1 - 4t_2 - 16t_3 + 5w_1 + 14w_2 \geq 0 \\
& -5t_1 - 7t_2 - 10t_3 + 8w_1 + 15w_2 \geq 0 \\
& -4t_1 - 9t_2 - 13t_3 + 7w_1 + 12w_2 \geq 0 \\
& 5w_1 + 14w_2 = 0 \\
& t_1 \geq 0.0001 \\
& t_2 \geq 0.0001 \\
& t_3 \geq 0.0001 \\
& w_1 \geq 0.0001 \\
& w_2 \geq 0.0001
\end{aligned}$$

Hospital 2 LP

$$\begin{aligned}
\max\ & z = 5t_1 + 7t_2 + 10t_3 \\
\text{s.t.}\ & -9t_1 - 4t_2 - 16t_3 + 5w_1 + 14w_2 \geq 0 \\
& -5t_1 - 7t_2 - 10t_3 + 8w_1 + 15w_2 \geq 0 \\
& -4t_1 - 9t_2 - 13t_3 + 7w_1 + 12w_2 \geq 0 \\
& 8w_1 + 15w_2 = 0 \\
& t_1 \geq 0.0001 \\
& t_2 \geq 0.0001 \\
& t_3 \geq 0.0001 \\
& w_1 \geq 0.0001 \\
& w_2 \geq 0.0001
\end{aligned}$$

Hospital 3 LP

$$\begin{aligned}
\max\ & z = 4t_1 + 9t_2 + 13t_3 \\
\text{s.t.}\ & -9t_1 - 4t_2 - 16t_3 + 5w_1 + 14w_2 \geq 0
\end{aligned}$$

$$-5t_1 - 7t_2 - 10t_3 + 8w_1 + 15w_2 \geq 0$$
$$-4t_1 - 9t_2 - 13t_3 + 7w_1 + 12w_2 \geq 0$$
$$7w_1 + 12w_2 = 0$$
$$t_1 \geq 0.0001$$
$$t_2 \geq 0.0001$$
$$t_3 \geq 0.0001$$
$$w_1 \geq 0.0001$$
$$w_2 \geq 0.0001$$