

對數函數作業

1. 計算下列各值：

$$(1) \ln \sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2} \ln e = \frac{1}{2}$$

$$(2) e^{\ln 5} = 5$$

$$(3) e^{3 \ln 2 - 2 \ln 5} = e^{\ln 2^3 - \ln 5^2} = e^{\ln \frac{2^3}{5^2}} = \frac{2^3}{5^2} = \frac{8}{25}$$

$$(4) \ln \frac{e^3 \sqrt{e}}{e^{\frac{1}{3}}} = \ln e^3 + \ln \sqrt{e} - \ln e^{\frac{1}{3}} = 3 \ln e + \frac{1}{2} \ln e - \frac{1}{3} \ln e = 3 + \frac{1}{2} - \frac{1}{3} = \frac{18+3-2}{6} = \frac{19}{6}$$

2. 將下各對數寫成 $\log_3 2$ 與 $\log_3 5$ 的表示式：

$$(1) \log_3 270 = \log_3 (2 \times 3^3 \times 5) = \log_3 2 + 3 \log_3 3 + \log_3 5 = 3 + \log_3 2 + \log_3 5$$

$$(2) \log_3 (2.5) = \log_3 \frac{5}{2} = \log_3 5 - \log_3 2$$

$$(3) \log_3 \left(\frac{64}{125} \right) = \log_3 64 - \log_3 125 = \log_3 2^6 - \log_3 5^3 = 6 \log_3 2 - 3 \log_3 5$$

3. 化簡下列各對數：

$$(1) \log_3 (x^5 y^{-2}) = \log_3 x^5 + \log_3 y^{-2} = 5 \log_3 x - 2 \log_3 y$$

$$(2) \ln(x^2 \sqrt{4-x^2}) = \ln x^2 + \ln(4-x^2)^{\frac{1}{2}} = 2 \ln x + \frac{1}{2} \ln(4-x^2) = 2 \ln x + \frac{1}{2} \ln(2-x) + \frac{1}{2} \ln(2+x)$$

$$(3) \ln \left[\frac{1}{x} + \frac{1}{x^2} \right] = \ln \frac{x+1}{x^2} = \ln(x+1) - 2 \ln x$$

$$(4) \ln(x^3 e^{-x^2}) = 3 \ln x - x^2$$

$$(5) \ln \left[\frac{\sqrt[4]{x}}{x^3 \sqrt{1-x^2}} \right] = \frac{1}{4} \ln x - 3 \ln x - \frac{1}{2} \ln(1-x^2) = -\frac{11}{4} \ln x - \frac{1}{2} \ln(1-x) - \frac{1}{2} \ln(1+x)$$

4. 求解下列各方程式：

$$(1) 2 = e^{0.06x}$$

$$\text{解：} \ln 2 = \ln e^{0.06x} = 0.06x \Rightarrow x = \frac{\ln 2}{0.06}$$

$$(2) 3 = 2 + 5e^{-4x}$$

$$\text{解：} 3 = 2 + 5e^{-4x} \Rightarrow 5e^{-4x} = 1 \Rightarrow e^{-4x} = \frac{1}{5} \Rightarrow \ln e^{-4x} = \ln \frac{1}{5} \Rightarrow -4x = -\ln 5 \Rightarrow x = \frac{\ln 5}{4}$$

$$(3) 5 = 3 \ln x - \frac{1}{2} \ln x$$

$$\text{解：} 5 = 3 \ln x - \frac{1}{2} \ln x = \frac{5}{2} \ln x \Rightarrow \ln x = 2 = \ln e^2 \Rightarrow x = e^2$$

$$(4) \ln x = \frac{1}{3}(\ln 16 + 2 \ln 2)$$

$$\text{解：} \ln x = \frac{1}{3}(\ln 16 + 2 \ln 2) = \frac{1}{3} \ln(16 \times 2^2) = \frac{1}{3} \ln 64 = \ln 64^{\frac{1}{3}} = \ln 4 \Rightarrow x = 4$$

$$(5) \frac{5}{1+2e^{-x}} = 3$$

$$\text{解：} \frac{5}{1+2e^{-x}} = 3 \Rightarrow 5 = 3 + 6e^{-x} \Rightarrow e^{-x} = \frac{1}{3} \Rightarrow -x = \ln \frac{1}{3} = -\ln 3 \Rightarrow x = \ln 3$$

5. 假設某銀行的年利率為 7%，採用連續複利計算，請問多久後金額變成本金的兩倍。

解：假設本金為 P ，則

$$2P = Pe^{(0.07)(t)} \Rightarrow e^{0.07t} = 2 \Rightarrow 0.07t = \ln 2 \Rightarrow t = \frac{\ln 2}{0.07}$$

6. 一樣本的鐳元素由 50 毫克衰退成 5 毫克時，經過幾年？(已知鐳的半衰期為 1690 年)

$$\text{解：} 5 = 50e^{-\frac{\ln 2}{1690}t} \Rightarrow e^{-\frac{\ln 2}{1690}t} = \frac{1}{10} \Rightarrow -\frac{\ln 2}{1690}t = \ln \frac{1}{10} = -\ln 10 \Rightarrow t = \frac{(\ln 10)(1690)}{\ln 2}$$