Fourier Series Synthesis

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※ 本例週期方波 $x(t)=\begin{cases}1, & 0<t\le 1\\ 0, & 1<t\le 2\end{cases}$

利用方程式 $x(t)=\frac{a_0}{2}+\sum_{n=1}^{12}c_n\cos\left[2\pi t\frac{n}{T_0}+\theta_n\right]$ 繪出波形

先找出c_n與θ_n

$c_n=\sqrt{a_n^{\,2}+b_n^{\,2}}$, $\theta_n=\text{-}\arctan\frac{b_n}{a_n}$

經計算後 …

$$\frac{a_0}{2}=\frac{1}{T}\int_0^2 x(t)dt=\frac{1}{2}$$

$$a_n=\frac{2}{T}\int_0^2 x(t)\cos\frac{2\pi t}{T}dt=0$$

$$b_n=\frac{2}{T}\int_0^2 x(t)\sin\frac{2\pi t}{T}dt=\int_0^1 1\cdot\sin n\pi t dt=-\frac{1}{n\pi}\cos n\pi t\Big|_0^1=\frac{1}{n\pi}(1-\cos n\pi)$$

$$=\begin{cases}\frac{2}{n\pi}, n=1,3,5,7,9,11...\\ 0\quad , n=2,4,6,8,10,12...\end{cases}$$

所以，各個係數爲以下所列：

$$\frac{a_0}{2}=\frac{1}{2}$$

$$a_1=0, b_1=\frac{2}{\pi}, c_1=\frac{2}{\pi}, \theta_1=1.5708$$

$$a_2=0, b_2=0, c_2=0, \theta_2=0$$

$$a_3=0, b_3=\frac{2}{3\pi}, c_3=\frac{2}{3\pi,}\theta_3=1.5708$$

$$a_4=0, b_4=0, c_4=0, \theta_4=0$$

$$a_5=0, b_5=\frac{2}{5\pi}, c_5=\frac{2}{5\pi}, \theta_5=1.5708$$

$$a_6=0, b_6=0, c_6=0, \theta_6=0$$

$$a_7=0, b_7=\frac{2}{7\pi}, c_7=\frac{2}{7\pi}, \theta_7=1.5708$$

$$a_8=0, b_8=0, c_8=0, \theta_8=0$$

$$a_9=0, b_9=\frac{2}{9\pi}, c_9=\frac{2}{9\pi}, \theta_9=1.5708$$

$$a_{10}=0, b_{10}=0, c_{10}=0, \theta_{10}=0$$

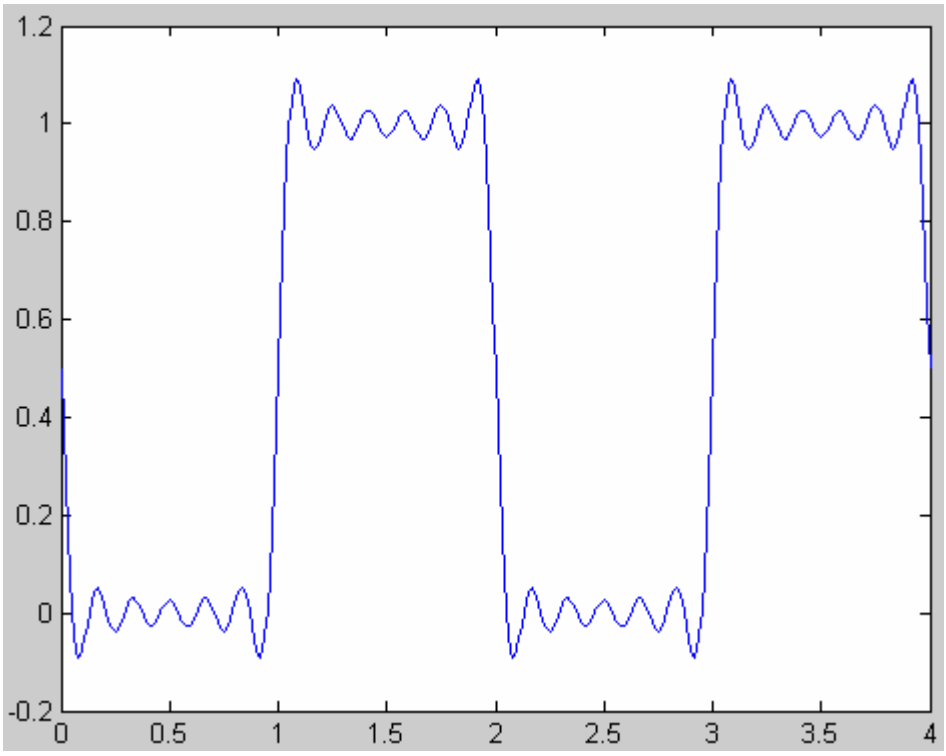
$$a_{11}=0, b_{11}=\frac{2}{11\pi}, c_{11}=\frac{2}{11\pi}, \theta_{11}=1.5708$$

$$a_{12}=0, b_{12}=0, c_{12}=0, \theta_{12}=0$$

將以上係數a_0、c_n與θ_n代入程式$x(t)=\frac{a_0}{2}+\sum_{n=1}^{12}c_n\cos\left[2\pi t\frac{n}{T_0}+\theta_n\right]$

於 MATLAB 鍵入以下指令：

```
>> t =0:0.01:4;
>>x=(1/2)+(2/pi)*cos(pi*t+1.5708)+(2/(3*pi))*cos(3*pi*t+1.5708)+(2/(5*pi))*cos(5
*pi*t+1.5708)+(2/(7*pi))*cos(7*pi*t+1.5708)+(2/(9*pi))*cos(9*pi*t+1.5708)+(2/(11
*pi))*cos(11*pi*t+1.5708);
>> plot(t,x)
```



執行出來的圖形，和例子時間對應之值相反(檢查過算式，應該沒錯，不知是哪打錯)！？

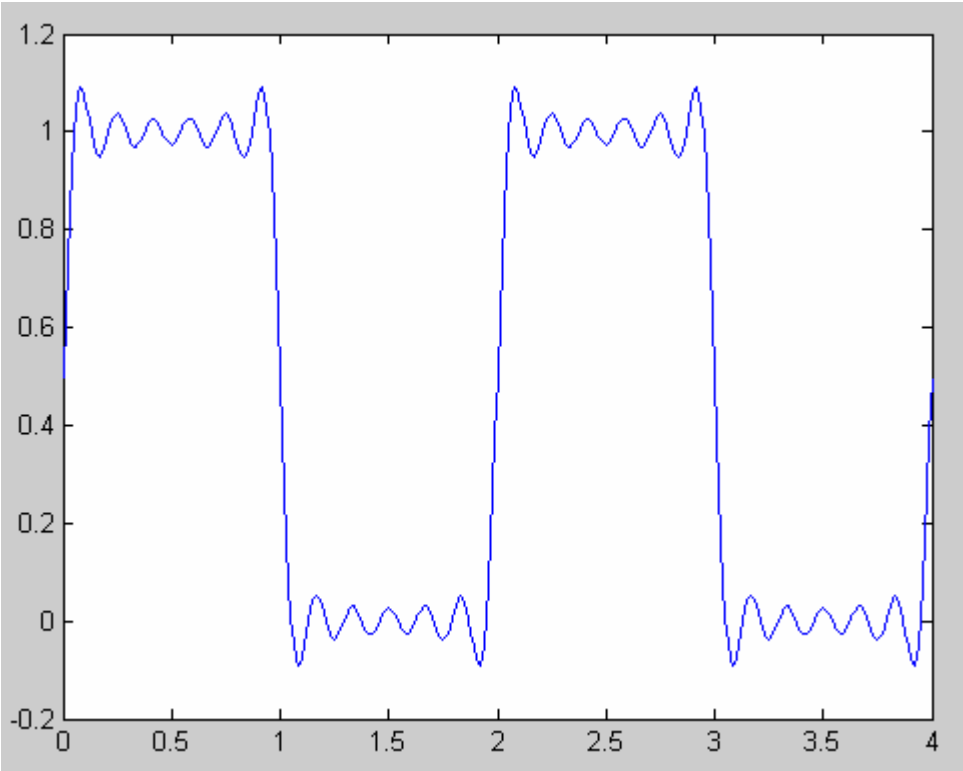
若以下列型式來執行

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{12} a_n \cos(2\pi t \frac{n}{T}) + b_n \sin(2\pi t \frac{n}{T})$$

代入係數a_0、a_n與b_n

如以下指令：

```
>>t =0:0.01:4;
>>x=(1/2)+(2/pi)*sin(pi*t)+(2/(3*pi))*sin(3*pi*t)+(2/(5*pi))*sin(5*pi*t)+(2/(7*pi))*sin(7*pi*t)+(2/(9*pi))*sin(9*pi*t)+(2/(11*pi))*sin(11*pi*t);
>>plot(t,x)
```



可以！

※ 本例週期方波 $x(t)=\begin{cases}-1, & -2<t\le 0\\ 1, & 0<t\le 2\end{cases}$

利用方程式 $x(t)=\frac{a_0}{2}+\sum_{n=1}^{12}c_n\cos\left[2\pi t\frac{n}{T_0}+\theta_n\right]$ 繪出波形

先找出c_n與θ_n

$c_n=\sqrt{a_n^{\,2}+b_n^{\,2}}$ ，$\theta_n=-\arctan\frac{b_n}{a_n}$

$\frac{a_0}{2}=0$

$$a_n = 0$$

$$b_n = \frac{2}{T/2}\int_0^{T/2} x(t)\sin\frac{2\pi t}{T}dt = \int_0^2 \sin\frac{n\pi t}{2}dt = \frac{2}{n\pi}(1-\cos n\pi)$$

$$\begin{cases} \frac{4}{n\pi}, n = 1,3,5,7,9,11... \\ 0 \quad , n = 2,4,6,8,10,12... \end{cases}$$

所以，各個係數爲以下所列：

$$\frac{a_0}{2} = 0$$

$$a_1 = 0, b_1 = \frac{4}{\pi}, c_1 = \frac{4}{\pi}, \theta_1 = 1.5708$$

$$a_2 = 0, b_2 = 0, c_2 = 0, \theta_2 = 0$$

$$a_3 = 0, b_3 = \frac{4}{3\pi}, c_3 = \frac{4}{3\pi,}\theta_3 = 1.5708$$

$$a_4 = 0, b_4 = 0, c_4 = 0, \theta_4 = 0$$

$$a_5 = 0, b_5 = \frac{4}{5\pi}, c_5 = \frac{4}{5\pi}, \theta_5 = 1.5708$$

$$a_6 = 0, b_6 = 0, c_6 = 0, \theta_6 = 0$$

$$a_7 = 0, b_7 = \frac{4}{7\pi}, c_7 = \frac{4}{7\pi}, \theta_7 = 1.5708$$

$$a_8 = 0, b_8 = 0, c_8 = 0, \theta_8 = 0$$

$$a_9 = 0, b_9 = \frac{4}{9\pi}, c_9 = \frac{4}{9\pi}, \theta_9 = 1.5708$$

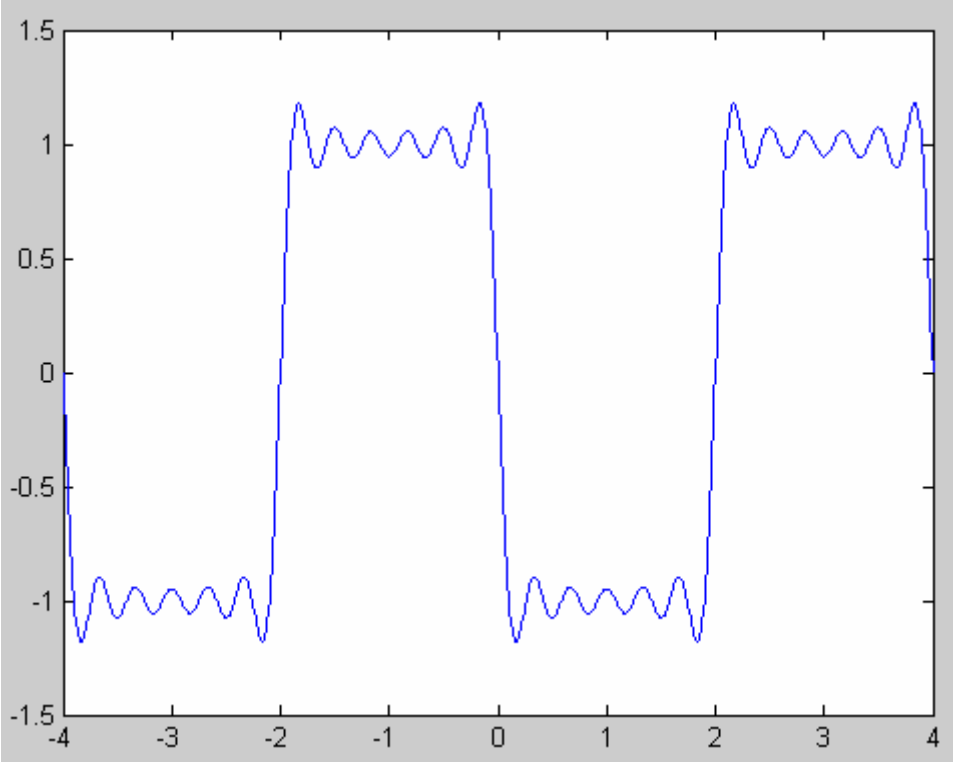
$$a_{10} = 0, b_{10} = 0, c_{10} = 0, \theta_{10} = 0$$

$$a_{11} = 0, b_{11} = \frac{4}{11\pi}, c_{11} = \frac{4}{11\pi}, \theta_{11} = 1.5708$$

$$a_{12} = 0, b_{12} = 0, c_{12} = 0, \theta_{12} = 0$$

```
>>t=-4:0.01:4;
>>x=(4/pi)*cos(((pi*t)/2)+1.5708)+(4/(3*pi))*cos(((3*pi*t)/2)+1.5708)+(4/(5*pi))*c
```

os(((5*pi*t)/2)+1.5708)+(4/(7*pi))*cos(((7*pi*t)/2)+1.5708)+(4/(9*pi))*cos(((9*pi*t)/2)+1.5708)+(4/(11*pi))*cos(((11*pi*t)/2)+1.5708);
>> plot(t,x)



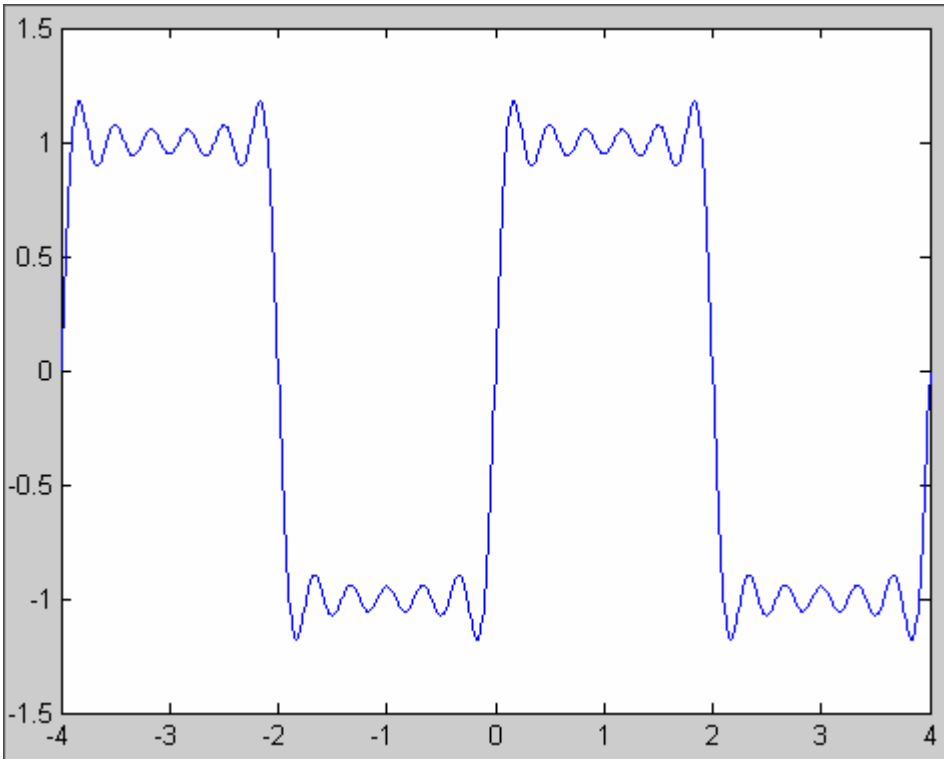
執行出的圖形，亦和上一例子有同樣的問題，不知哪個步驟出錯了！

若以下列型式來執行

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{12} a_n \cos(2\pi t \frac{n}{T}) + b_n \sin(2\pi t \frac{n}{T})$$

如以下指令：

>>t=-4:0.01:4;
>>x=(4/pi)*sin((pi*t/2))+(4/pi)*(1/3)*sin((3*pi*t/2))+(4/pi)*(1/5)*sin((5*pi*t/2))+(4/pi)*(1/7)*sin((7*pi*t/2))+(4/pi)*(1/9)*sin((9*pi*t/2)) +(4/pi)*(1/11)*sin((11*pi*t/2));
>>plot(t,x)



可以！

※ 再試一例子 $x(t)=\begin{cases} x+2, & -2<t\le 0 \\ 1, & 0<t\le 2 \end{cases}$

利用方程式 $x(t)=\frac{a_0}{2}+\sum_{n=1}^{12} c_n \cos\left[2\pi t \frac{n}{T_0}+\theta_n\right]$ 繪出波形

先找出c_n與θ_n

$c_n=\sqrt{a_n^{\,2}+b_n^{\,2}}$ ，$\theta_n=\text{-arctan}\frac{b_n}{a_n}$

$$\frac{a_0}{2}=\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)dt=\frac{1}{4}\int_{-2}^{2} x(t)dt=\frac{1}{4}\left\{\int_{-2}^{0}(x+2)dt+\int_{0}^{2}dt\right\}=1$$

$$a_n = \frac{1}{T/2}\int_{\frac{-2}{T}}^{\frac{2}{T}} x(t)\cos\frac{n\pi}{2}tdt = \frac{1}{2}\left\{\int_{-2}^{0}(x+2)\cos\frac{n\pi}{2}tdt + \int_{0}^{2}\cos\frac{n\pi}{2}tdt\right\} = \frac{2}{n^2\pi^2}(1-\cos n\pi)$$

$$= \begin{cases} \frac{4}{n^2\pi^2} & n = 1,3,5\cdots \\ 0 & n = 2,4,6\cdots \end{cases}$$

$$b_n = \frac{2}{T/2}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)\sin\frac{n\pi}{2}tdt = \frac{1}{2}\left\{\int_{-2}^{0}(x+2)\sin\frac{n\pi}{2}tdt + \int_{0}^{2}\sin\frac{n\pi}{2}tdt\right\} = -\frac{1}{n\pi}(1+\cos n\pi)$$

$$= \begin{cases} -\frac{2}{n\pi} & n = 1,3,5\cdots \\ 0 & n = 2,4,6\cdots \end{cases}$$

所以，各個係數爲以下所列：

$$\frac{a_0}{2} = 1$$

$$a_1 = \frac{4}{\pi^2}, b_1 = 0, c_1 = \frac{4}{\pi^2}, \theta_1 = 0$$

$$a_2 = 0, b_2 = -\frac{1}{\pi}, c_2 = \frac{1}{\pi}, \theta_2 = 1.5708$$

$$a_3 = \frac{4}{9\pi^2}, b_3 = 0, c_3 = \frac{4}{9\pi^2}\theta_3 = 0$$

$$a_4 = 0, b_4 = -\frac{1}{2\pi}, c_4 = \frac{1}{2\pi}, \theta_4 = 1.5708$$

$$a_5 = \frac{4}{25\pi^2}, b_5 = 0, c_5 = \frac{4}{25\pi^2}, \theta_5 = 0$$

$$a_6 = 0, b_6 = -\frac{1}{3\pi}, c_6 = \frac{1}{3\pi}, \theta_6 = 1.5708$$

$$a_7 = \frac{4}{49\pi^2}, b_7 = 0, c_7 = \frac{4}{49\pi^2}, \theta_7 = 0$$

$$a_8 = 0, b_8 = -\frac{1}{4\pi}, c_8 = \frac{1}{4\pi}, \theta_8 = 1.5708$$

$$a_9 = \frac{4}{81\pi^2}, b_9 = 0, c_9 = \frac{4}{81\pi^2}, \theta_9 = 0$$

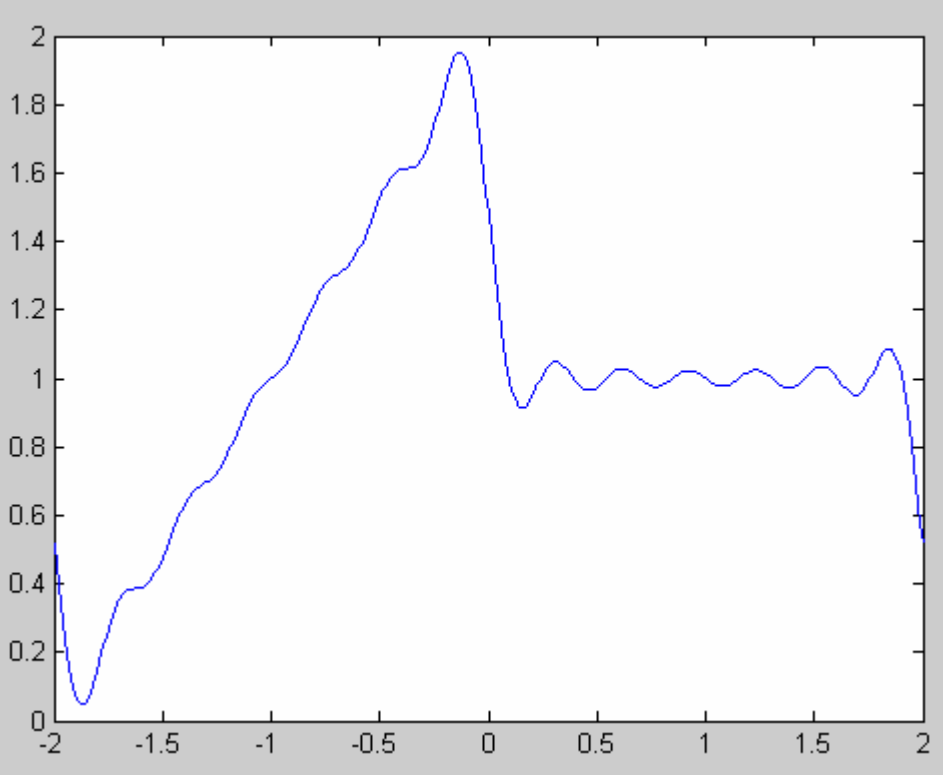
$$a_{10} = 0, b_{10} = -\frac{1}{5\pi}, c_{10} = \frac{1}{5\pi}, \theta_{10} = 1.5708$$

$$a_{11} = \frac{4}{121\pi^2}, b_{11} = 0, c_{11} = \frac{4}{121\pi^2}, \theta_{11} = 0$$

$$a_{12} = 0, b_{12} = -\frac{1}{6\pi}, c_{12} = \frac{1}{6\pi}, \theta_{12} = 1.5708$$

將係數代入方程式，執行以下指令：

```
>> t=-2:0.01:2;
>>x=1+(4/pi^2)*cos((pi*t)/2)+(1/pi)*cos(pi*t+1.5708)+(4/(9*pi^2))*cos((3*pi*t)/2)+
(1/(2*pi))*cos(2*pi*t+1.5708)+(4/(25*pi^2))*cos((5*pi*t)/2)+(1/(3*pi))*cos(3*pi*t+
1.5708)+(4/(49*pi^2))*cos((7*pi*t)/2)+(1/(4*pi))*cos(4*pi*t+1.5708)+(4/(81*pi^2))
*cos((9*pi*t)/2)+(1/(5*pi))*cos(5*pi*t+1.5708)+(4/(121*pi^2))*cos((11*pi*t)/2)+(1/
(6*pi))*cos(6*pi*t+1.5708);
>> plot(t,x)
```



差不多趨於想要的波形！