

5.1 Consider a particle that can be in only three states, with energies -0.05 , 0 , and 0.05 eV. This particle is in equilibrium with a reservoir at 300 K.

- (a) Calculate the partition function for this particle.
- (b) Calculate the probability for this particle to be in each state.
- (c) Since the zero point for measuring energies is arbitrary, one can assign the three states as 0 , $+0.05$, and $+0.10$ eV instead. Repeat (a) and (b).

5.2 Consider a property E , which may be energy, entropy...etc. The deviation of the property of a microstate i from its average is defined as $\Delta E_i = E_i - \langle E \rangle$. The root-mean-square (rms) deviation, or standard deviation, σ_E , is defined as

$$\sigma_E^2 = \langle \Delta E_i^2 \rangle.$$

(a) Show that $\sigma_E^2 = \langle E^2 \rangle - \langle E \rangle^2$.

(b) If the property is U , prove that $\langle U^2 \rangle = \frac{1}{Q} \frac{\partial^2 Q}{\partial \beta^2}$.

(c) The heat capacity is defined as $C \equiv \partial \langle U \rangle / \partial T$. Show that $\sigma_U = kT \sqrt{C/k}$.